BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 1, January 1977

COMPLEX GEOMETRY AND OPERATOR THEORY[1]

BY M. J. COWEN[2] AND R. G. DOUGLAS

Communicated by I. M. Singer, September 1, 1976

One of the principal goals of spectral theory for operators is to find unitary invariants which are local relative to the spectrum. Multiplicity theory provides a complete set of such invariants for normal operators on (complex) Hilbert space. For general operators on finite-dimensional Hilbert space a nilpotent operator is attached to each point of the spectrum and these "local operators" together with their relative location provide a complete set of unitary invariants. In this note we announce analogous results for a class of operators whose characteristic property is having an open set of eigenvalues. Included in this class are the backward shift together with the adjoint of various subnormal, hyponormal, and weighted shift operators. Although our goal is to provide a systematic spectral theoretic approach to the study of this type of operator, here we are concerned only with a result on unitary equivalence.

For Ω a connected open subset of $\mathbf{C}$ and n a positive integer, let $\mathcal{B}_n(\Omega)$ denote the (bounded linear) operators T defined on the separable Hilbert space $\mathcal{H}$ which satisfy: (1) Ω is contained in the spectrum $\sigma(T)$; (2) $(T - \omega)\mathcal{H} = \mathcal{H}$ for ω in Ω; (3) $\dim \ker(T - \omega) = n$ for ω in Ω; and (4) $\bigvee_{\omega \in \Omega} \ker(T - \omega) = \mathcal{H}$. To study T in $\mathcal{B}_n(\Omega)$ we introduce the "local operators" N_ω on $\mathcal{N}_\omega$ defined for each ω in Ω, where $\mathcal{N}_\omega = \ker(T - \omega)^{n+1}$ and $N_\omega = (T - \omega)|\mathcal{N}_\omega$. Observe that N_ω is nilpotent of order $n + 1$ on a space of dimension $n(n + 1)$. Our principal result is

THEOREM 1. *Operators T and T' in $\mathcal{B}_n(\Omega)$ are unitarily equivalent if and only if N_ω is unitarily equivalent to N'_ω for ω in Ω.*

REMARKS. (1) For $n = 1$ the real analytic function $\operatorname{tr} N_\omega N_\omega^*$ is a complete set of unitary invariants. In general, the complete set of unitary invariants can be chosen to be real analytic functions defined as the traces of a finite number of words in N_ω and N_ω^*.

(2) For "generic operators" one need only consider $(T - \omega)|\ker(t - \omega)^3$.

(3) Comparatively trivial results obtain if one allows knowledge of either $(T - \omega)|\ker(T - \omega)^k$ or $T|\bigvee_{i=1}^k \ker(T - \omega_i)$ with $\omega_1, \omega_2, \ldots, \omega_k$ in Ω for arbitrary k.

AMS (MOS) subject classifications (1970). Primary 47A10, 47B20, 53B35.

[1] Supported by grants from the National Science Foundation.

[2] Sloan Foundation Fellow and SUNY-Research Foundation Fellow.