BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 1, January 1977

MODULI OF VECTOR BUNDLES ON CURVES WITH PARABOLIC STRUCTURES

BY C. S. SESHADRI

Communicated by Stephen S. Shatz, April 8, 1976

Let H be the upper half plane and Γ a discrete subgroup of Aut$H$. Suppose that $H \bmod \Gamma$ is of *finite measure*. This work stems from the question whether there is an algebraic interpretation for the moduli of unitary representations of Γ similar to the case when $H \bmod \Gamma$ is *compact* (cf. [3], [4], [5]). We show that this is indeed the case via the moduli of vector bundles on the compactification of $H \bmod \Gamma$, provided with some additional structures which we propose to call *parabolic structures*. The idea of parabolic structures is inspired from A. Weil's work [6, §2, Chapter I, p. 56].

Let X be a *smooth, irreducible, projective curve* defined, say, over an algebraically closed field k. By *vector bundles* on X we understand algebraic vector bundles.

DEFINITION 1. Let V be a vector bundle on X and $Q \in X$. Then a *quasi-parabolic structure of V at Q* is giving a flag on the fibre V_Q of V at Q, i.e., giving linear subspaces F^iV_Q of V_Q,

$$V_Q = F^1V_Q \supset F^2V_Q \supset \cdots \supset F^rV_Q; \quad \dim F^iV_Q = l_i; \qquad l_1 > l_2 > \cdots > l_r.$$

We call $l = (l_1, \ldots, l_r)$ the *type* (or flag type) of the quasi-parabolic structure. Let $k_1 = l_1 - l_2, k_2 = l_2 - l_3, \ldots, k_{r-1} = l_{r-1} - l_r, k_r = l_r$; then k_i are called the *multiplicities* of the quasi-parabolic structure.

DEFINITION 2. Let V be a vector bundle on X and $Q \in X$. Then a *parabolic structure of V at Q* is giving

(i) a quasi-parabolic structure of V at Q; say $l = (l_1, \ldots, l_r)$ is its type and $\{k_i\}$ its multiplicities, and

(ii) constants $\alpha = (\alpha_1, \ldots, \alpha_n)$ called the *weights* of the parabolic structure such that $0 \leqslant \alpha_1 \leqslant \alpha_2 \leqslant \cdots \leqslant \alpha_n < 1$ and there are r distinct elements among α, say $\alpha' = (\alpha'_1, \ldots, \alpha'_r)$, $0 \leqslant \alpha'_1 < \alpha'_2 < \cdots < \alpha'_r < 1$, such that α'_1 occurs k_1 times, α'_2 occurs k_2 times, $\ldots$, α'_r occurs k_r times among α. We call *α'_i the weight of F^iV_Q*. Note that $l_1 = n = rkV$.

Let V, W be vector bundles on X with *quasi-parabolic* structures at Q. An isomorphism f: $V \longrightarrow W$ of vector bundles is said to be a *quasi-parabloic isomorphism* if the types of V, W at Q are the same and $f_Q(F^iV_Q) = F^iW_Q$ (f_Q:

AMS (MOS) subject classifications (1970). Primary 14H20, 14H99; Secondary 32G13.