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SUBSEQUENCES OF SEQUENCES OF RANDOM VARIABLES

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Chatterji [2] has formulated the following heuristic principle: given any limit property for independent identically distributed random variables (i.i.d.r.v.'s), there exists an analogous property such that an arbitrary sequence of r.v.'s always has some subsequence possessing this analogous property. By 'arbitrary', we mean that no assumption concerning dependence is made, though it may be necessary to assume moment conditions on the r.v.'s. The purpose of this note is to announce Theorem 1, which makes this principle precise in the case where the property is an "a.s. limit theorem", a concept we formalise below.

Let $P(R)$ denote the space of probability measures on R. For $\Pi \in P(R)$, let

$$\begin{aligned} \alpha(\Pi) &= \infty && \text{if } \int |x| \Pi(dx) = \infty, \\ &= \int x \ \Pi(dx) && \text{otherwise,} \end{aligned}$$

and let $\beta(\Pi) = \int (x - \alpha(\Pi))^2 \Pi(dx)$.

Define a *statute* to be a measurable subset A of $P(R) \times R^\infty$ such that, whenever $\Pi \in P(R)$ and $\{X_n\}$ is i.i.d. with distribution Π, then $(\Pi, X_1(\omega), X_2(\omega), \ldots) \in A$ a.s. For example, the statutes corresponding to the strong law of large numbers and the law of the iterated logarithm are

$$A_1 = \left\{ (\Pi, \mathbf{x}) \colon \alpha(\Pi) = \infty \text{ or } \lim_{n \to \infty} \sum_{i=1}^{n} x_i \Big/ n = \alpha(\Pi) \right\}.$$

$$A_2 = \left\{ (\Pi, \mathbf{x}) \colon \beta(\Pi) = \infty \text{ or } \limsup_{n \to \infty} \frac{\sum_{i=1}^{n} x_i - n\alpha(\Pi)}{(2n \log \log(n))^{1/2}} = \beta^{1/2}(\Pi) \right\}.$$

Here $\mathbf{x} = (x_i)$ denotes a generic point in R^∞.

It seems clear that any "a.s. limit theorem for i.i.d.r.v.'s" may be represented by a statute. We need the following technical condition, which is satisfied by the statutes corresponding to most nontrivial such theorems.

(1) If $(\Pi, \mathbf{x}) \in A$ and $\mathbf{x}'$ satisfies $\Sigma |x_i - x_i'| < \infty$, then $(\Pi, \mathbf{x}') \in A$.

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