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Unitary representations and harmonic analysis, an introduction, by Mitsuo Sugiura, Kodansha Ltd., Tokyo; John Wiley & Sons, New York, London, Sydney, Toronto, 1975, xiii + 402 pp., $34.00.

There is the story of the cellist, playing but a single note, who explained to his one friend and many enemies that, while they sought the golden sound, he had found it. Just so, those mathematicians penetrating the mysteries of harmonic analysis may well say to all the rest that while others seek, we have found the center of mathematical elegance.

It is, of course, well known to astrologers, economists, and those of us who plant seeds during the dark of the moon that everything goes in cycles, yet it was left for that *paure orphelin* Jean Baptiste Joseph Fourier to establish the matter beyond any shadow of doubt. Unfortunately his discoveries were so disconcerting as to cast misgivings on their utility for prediction. Everything goes in cycles, but the manner of its going changes from interval to interval and moment to moment in unsatisfactory ways. Yet, since Fourier's time, harmonic analysis has intersected most all mathematical problems short of that of forecasting the future. One establishes this fact both empirically and intellectually. The empirical proof consists in compiling even a minibiography whose length will be exceeded only by the brilliance of some of the contributors. The intellectual argument is simpler. Quote the words of Dieudonné spoken during the American Academy Workshop on the Evolution of Modern Mathematics to the effect: "–it is strange to study the work of Harish-Chandra in the last 15 years on representations of semisimple Lie groups. He uses such a fantastic arsenal of techniques taken from all over mathematics. It is quite clear that the number of people who are able to understand this work is very small at present, because it taxes the intellectual capacity of a person."

In defiance of the gods, Sugiura has written a fine book for mere mortals. Unfortunately one senses that it represents a good idea whose time has passed–the tide has come and gone, more than once, before its arrival. This feeling has a number of parts: First, the functional analysis required for its easy comprehension has been replaced in many quarters by an amorphous subject governed largely by topological interests and considerations. Second, there have been a large number of books and papers written on representation theory during the last decade, even during the last few years. Some of these are very good. Third, and most unfortunate of all, the long marriage of mathematics and physics seems destined for a final separation. We discuss these points in sequence.

Turning to the first, much of the functional analysis used by Sugiura is of a classical nature, according to a definition of Singer, having been around for more than ten years. In his introductory chapter, the author makes a very natural transition from the complex representation theory of a finite group to that of a compact group by means of the spectral theorem for a bounded selfadjoint operator on a Hilbert space, by a brief use of commutants and von Neumann algebras, and by a final appeal to the direct integral of Hilbert spaces. This indicates the level of soft analysis that Sugiura requires in his