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NEW VIEWPOINTS IN THE GEOMETRY OF SUBMANIFOLDS OF R^N

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0. Introduction. The geometry of submanifolds of euclidean space is the oldest branch of differential geometry. The subject was the original source of most of the classical and modern ideas in the field, and still is the setting in which seemingly complicated general phenomena are most easily understood. In fact as Allendoerfer [1] once said "... an excellent way of discovering [theorems] is first to consider an imbedded manifold and then later to invent a proof applicable to a general abstract manifold." In fact it is often true that a concept which can be defined for submanifolds of Riemannian manifolds may be introduced in a simpler and more natural way for submanifolds immersed in euclidean space. In this paper we will always try to adopt the second course.

The geometry of hypersurfaces, that is, codimension one immersions, is familiar ground to most mathematicians, but once the codimension is increased the material is neither as familiar nor as satisfactory. In fact rather mysterious concepts appear, such as E. Cartan's exterior orthogonality of a linear system of quadratic forms, and C. Allendoerfer's type number. The plan of this report is to systematically expose the basic invariants of arbitrary codimension immersions in a way we regard as algebraically and geometrically natural. The viewpoints are often not standard. New invariants such as curvature deficiency and the tth-trace rank are introduced and applied. Some old invariants such as the Allendoerfer type number are given new geometric interpretations, and old applications are reexamined and refined. New methods such as the systematic exploitation of the minimal enveloping subspaces of linear systems in tensor and exterior products lead to new viewpoints in which the Gauss integrability conditions appear as a short exact sequence of linear systems. Our viewpoint is both algebraically and geometrically natural, the symbiotic effect of this double viewpoint will be illustrated through the Allendoerfer theory of type number and some recent results on reducibility of immersions.

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