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FAILURE OF A QUADRATIC ANALOGUE OF SERRE'S CONJECTURE

BY S. PARIMALA

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Let A be a commutative ring with identity. By an *inner product A-space* we shall understand, as in [6], a pair (P, q), where P is a finitely generated projective A-module and q is a symmetric bilinear form $P \times P \longrightarrow A$ which is nonsingular (i.e. induces an isomorphism $P \xrightarrow{\sim} P^*$). If B is a commutative A-algebra we obtain an inner product B-space $(B \otimes_A P, B \otimes_A q)$. Inner product B-spaces isomorphic to one of these will be said to be *extended* from A.

The quadratic analogue of Serre's conjecture is the affirmation of:

Suppose A is a polynomial algebra $K[X_1, \ldots, X_n]$ over a field K. Is every inner product A-space extended from K?

This question is motivated by the following evidence.

(1) Serre's conjecture that projective A-modules are free, hence extended from K, has recently been proved by Quillen and Suslin (cf. [4]). Moreover this immediately implies that "symplectic A-spaces" are extended from K (see e.g. [1, Chapter IV, (4.11.2)]).

(2) If $\mathrm{Char}(K) \neq 2$ then a theorem of Karoubi [7, Theorem 1.1] implies that every inner product A-space is *stably isomorphic* to one extended from K.

(3) A theorem of Harder (see [8, Theorem 13.4.3]) gives an affirmative response to (QS) for $n = 1$.

A major tool in Quillen's proof of Serre's conjecture is:

QUILLEN'S LOCALIZATION THEOREM [11]. *Let A be a commutative ring, let T be an indeterminate, and let M be a finitely presented $A[T]$-module. If, for all maximal ideals $\mathfrak{m}$ of A, $M_{\mathfrak{m}}$ is extended from $A_{\mathfrak{m}}$, then M is extended from A.*

(4) The analogue of Quillen's localization theorem for inner product spaces has been proved in [3].

The other main tool Quillen uses is:

HORROCK'S THEOREM [5]. *Let A be a local ring and let P be a finitely generated projective $A[T]$-module. If P extends to a locally free sheaf on $\mathbf{P}_A^1$, then P is extended from A (hence free).*

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