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BOUNDED MEAN OSCILLATION WITH ORLICZ NORMS AND DUALITY OF HARDY SPACES

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This paper is a summary of the contents in [5] (with the same title).

BMO was introduced by John and Nirenberg as the space of locally integrable functions f on $\mathbf{R}^n$ such that

$$\int_Q |f(x) - f_Q|\, dx \leqslant Am(Q) \tag{1}$$

for some constant f_Q and for all cubes Q in $\mathbf{R}^n$, where $A \geqslant 0$ only depends on f. (m is the Lebesgue measure in $\mathbf{R}^n$.) John and Nirenberg [2] have shown that (1) will imply

$$m\{x \in Q;\ |f(x) - f_Q| > t\} \leqslant C_1 m(Q) \exp(-C_2 t/A), \quad t > 0, \tag{2}$$

with A the same as in (1).

If we assume that f a priori only is a measurable function on $\mathbf{R}^n$ and replace (1) by

$$m\{x \in Q;\ |f(x) - f_Q| > A'\} < \tfrac{1}{2} m(Q), \tag{1'}$$

then it is still true that $f \in BMO$ and (1) holds with $A \leqslant CA'$. In fact, a proof similar to that of [2] shows that (1)$'$ implies (2). If the constant 1/2 in (1)$'$ is replaced by a larger number, the result is no longer true.

Let us now assume that the numbers A and A' in (1) resp. (1)$'$ are not uniform but depend on x (but not on Q containing x). We define $M^\# f(x)$ resp. $M_0^\# f(x)$ as the infimum of such $A(x)$ resp. $A'(x)$. (2) will now be replaced by

$$\begin{aligned} &m\{x \in Q;\ |f(x) - f_Q| > t\} \\ &\qquad \leqslant C_1 m(Q) \exp(-C_2 t/s) + m\{x \in Q;\ M_0^\# f(x) > s\}, \quad s, t > 0. \end{aligned} \tag{3}$$

From (3) it follows that $\|M_0^\# f\|_{L^p}$ is equivalent to $\|M^\# f\|_{L^p}$, $1 < p < \infty$. It was shown by Fefferman and Stein in [1] (under some nonessential restrictions) that

$$\|Mf\|_{L^p} \leqslant C \|M^\# f\|_{L^p} \quad \text{modulo constants, } 1 < p < \infty, \tag{4}$$

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