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DYNAMICAL SYSTEMS ON HOMOGENEOUS SPACES[1]

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Consider the following class of dynamical systems: Let $X = G/\Gamma$ be the homogeneous space of a Lie group G where Γ is a lattice; i.e. a discrete subgroup of G such that G/Γ carries a finite G-invariant Borel measure μ. Let T be an affine transformation (AT) of G/Γ i.e. $T = T_g \circ \bar{A}$ where $\bar{A}$ is the quotient of a continuous group automorphism A of G such that $A\Gamma \subset \Gamma$ and T_g is the (left) translation of G/Γ by an element $g \in G$. If the differential dA of the automorphism is of determinant ± 1 then T is a measure-preserving transformation (MPT) of X; i.e. a dynamical system. More generally one may also consider groups of AT's, a typical case being a group of translation (i.e. $\bar{A} = \mathrm{Id}$) defined by a subgroup H of G. These systems together with their factors include many classical dynamical systems, e.g. toral automorphisms, geodesic and horocycle flows on a surface of constant negative curvature and, more generally, the G-induced Anosov systems on infra-homogeneous spaces (cf. [7]).

The purpose of this note is to announce various ergodic and other dynamical properties of AT's, and more generally of groups of AT's, and discuss their applications. Analogues of these results have also been proved for the case when G is the group of k-rational points of an algebraic group defined over a nondiscrete locally compact field k. However for simplicity here we confine ourselves to homogeneous spaces of Lie groups. The general statements as well as proofs of the results will appear elsewhere [2].

1. **Bernoulli shifts.** We have the following theorem.

THEOREM 1. *Let $T = T_g \circ \bar{A}$ be an AT of G/Γ and $\sigma = \mathrm{Ad}\, g \circ dA$ be the induced automorphism of the Lie algebra $\mathfrak{g}$ of G* (Ad *denotes the adjoint action of G on $\mathfrak{g}$). Assume that*

(i) *the restriction of σ to the maximal σ-invariant subspace on which all eigenvalues (possibly complex) are of unit absolute value, is a semisimple linear transformation and*

(ii) *T is a Kolmogorov automorphism.*

Then T is a Bernoulli shift.

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[1] Details of proofs will appear in the Journal of the Indian Mathematical Society.