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QUADRATIC PAIRS FOR ODD PRIMES

BY C. Y. HO

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The purpose of this note is to announce a result in the classification of the quadratic pair for odd prime where the group is quasisimple. The complete proof can be found in [4] and [5]. For the convenience of the reader the terminology particular to this area is recollected here. A finite group, X, is quasisimple if X is perfect and $X/Z(X)$ is simple, where $Z(X)$ is the center of X. Let M be a vector space over the field K and G a subgroup of the general linear group of M. Let $Q = \{g \in G\backslash\{1\} | M(g-1)^2 = 0\}$. We say that (G, M) is a quadratic pair if M is an irreducible KG module and G is generated by Q. Let p be an odd prime. We say that (G, M) is a quadratic pair for p if K is the field of p elements and the dimension of M over K is finite. Let $d = \min_{\sigma \in Q}\{\dim M(\sigma - 1)\}$ and $Q_d = \{\tau \in Q | \dim M(\tau - 1) = d\}$. For each $\sigma \in Q_d$, let $E(\sigma) = \{\tau \in Q | M(\sigma - 1) = M(\tau - 1)$ and $C_M(\sigma) = C_M(\tau)\} \cup \{1\}$. Then $E(\sigma)$ is an elementary abelian p-subgroup of G. Let $\Sigma = \{E(\sigma) | \sigma \in Q_d\}$. We say that (G, M) is a quadratic pair for 3 whose root group has order 3 if $|E| = 3$ for any $E \in \Sigma$.

Lemma 4.1 of [3] gives the following result. Let (G, M) be a quadratic pair for 3 whose root group has order 3. Let $\sigma, \tau \in Q_d$. Then $\langle \sigma, \tau \rangle$ is isomorphic to one of the following groups: (a) $SL(2, 3)$, (b) $SL(2, 5)$, (c) $SL(2, 3) \times Z_3$, (d) Z_3 or $Z_3 \times Z_3$, (e) the nonabelian 3-group of order 27, exponent 3 and nilpotent class 2.

THEOREM. *Let (G, M) be a quadratic pair for p, p odd, such that G is quasisimple. If (G, M) is a quadratic pair for* 3 *whose root group has order* 3, *then we also assume that for some $E \in \Sigma$, the set $\{F | F \in \Sigma$ and $\langle E, F \rangle \cong SL(2, 3) \times Z_3\}$ is empty. Under these conditions $G/Z(G)$ is isomorphic to one of the following groups.*

(1) *Groups of Lie type of odd characteristic*: $A_n(q)$ ($n \geq 2$ *except in the case* $q = 3$ *where we have* $n \geq 3$), ${}^2A_n(q)$ ($n \geq 2$), $B_n(q)$ ($n \geq 3$), $C_n(q)$ ($n \geq 2$), $D_n(q)$ ($n \geq 3$), ${}^2D_n(q)$ ($n \geq 3$), ${}^3D_4(q)$, $G_2(q)$, $F_4(q)$, $E_6(q)$, ${}^2E_6(q)$, $E_7(q)$ *where* $q = p^b$ *for some positive integer* b.

(2) *Alternating groups*: A_n, $n \geq 5$.

(3) *Groups of Lie type of even characteristic*: $PGU_n(2)$, $Sp(6, 2)$, $D_4(2)$, $G_2(4)$.

(4) *Sporadic groups*: *HJ*, *Suz*, Co_1.

Furthermore we have $p = 3$ *whenever* (2), (3) *or* (4) *holds.*

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