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*Monotonie: Lösbarkeit und Numerik bei Operatorgleichungen*, by Erich Bohl, Springer Tracts in Natural Philosophy, vol. 25, Springer-Verlag, Berlin, Heidelberg, New York, 1974, ix + 255 pp., \$26.20.

*Iterative methods for the solution of a linear operator equation in Hilbert space—A survey*, by Walter Mead Patterson, 3rd, Lecture Notes in Mathematics, vol. 394, Springer-Verlag, Berlin, Heidelberg, New York, 1974, 183 pp., \$8.20.

The books by Bohl and Patterson are detailed expositions of iterative methods for the solution of operator equations. The common thread between them is tenuous; the two books are surprisingly dissimilar in the point of view, scope and intent. The "Bibliography" in Patterson (which is intended to be complete with respect to the current literature for the topics that are covered) contains 145 references, while the "Literaturverzeichnis" in Bohl has 165 references. The intersection? Four standard textbooks in functional analysis (primarily for background material) and one paper: Kantorovič [13]. This 1939 paper (which was one of the first to realize the power of functional analysis methods in developing and unifying the theory of iterative methods) is certainly the beginning of the thin common thread in the books under review. In the spirit of the reviews in this Bulletin, a multiple review is expected to be a review of the "span" of the two books (rather than the union of two almost disjoint reviews). To put this "span" in a proper perspective, we first examine briefly some historic episodes and contemporary aspects of the broader area to which the two books belong. This is the field of numerical analysis in abstract spaces (alas, numerical functional analysis, abstract numerical analysis, or other names).

Numerical analysis and the traditional methods of computation have undergone structural changes over the past two decades. They are continuously being influenced by two definite trends, both of which received impetus, if not originated, about 1947–1948. The first is represented by the applications of abstract methods to some areas of numerical analysis. The second comes from computers and their logical structures. In this review, we shall be concerned only with the first trend.

In 1948, L. V. Kantorovič [14] published a fundamental long paper entitled *Functional analysis and applied mathematics*. This paper marks the beginning—and remains a landmark—in abstract numerical analysis. The thrust of the paper is to show explicitly that "the ideas and methods of functional analysis may be used for the construction and analysis of effective practical algorithms for the solution of mathematical problems with just as much success as has attended their use for the theoretical investigation (i.e., existence, uniqueness, etc.) of these problems. Moreover, results and error estimates from such a general point of view may in certain cases prove to be more complete and exact than those obtained for the separate special cases".

The beginning of modern numerical analysis took place about 1947. This was the date of the celebrated von Neumann-Goldstine paper [25]. About this time, Turing published a companion paper on roundoff error. (Inciden-