BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 5, September 1976

MULTIVARIABLE EXPANSION OF SOLUTIONS OF LINEAR EQUATIONS WITH SLOWLY VARYING COEFFICIENTS

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Communicated by Alberto P. Calderón, April 8, 1976

0. Introduction. This note concerns the study of the equation

$$L_{SV}[y(t;\epsilon)] \equiv a(\epsilon t;\epsilon)\frac{d^2y}{dt^2} + \epsilon b(\epsilon t;\epsilon)\frac{dy}{dt} + c(\epsilon t;\epsilon)y = 0, \tag{1}$$

subject to the initial conditions

$$y(0;\epsilon) = \gamma, \qquad y'(0;\epsilon) = \delta \tag{2}$$

where ϵ is a small parameter such that $0 \leqslant \epsilon \leqslant \epsilon_0$ for some $\epsilon_0 > 0$, while γ and δ are constants independent of ϵ. We present a method for multivariable expansions of the solutions of this problem which is a generalization of the technique employed by E. L. Reiss [1] in the treatment of the problem

$$L[y(t;\epsilon)] \equiv y'' + 2\epsilon y' + y = 0, \qquad t > 0 \quad (' = d/dt),$$

$$y(0;\epsilon) = 0, \qquad y'(0;\epsilon) = 1.$$

An important contribution of our study is the discovery of a method for systematically generating time scales from the differential equation itself.

1. Derivation of time scales. Consider an interval D which includes $t = 0$ and impose the following conditions:

(C.1) The functions a, b, c are analytic in D.

(C.2) The functions a, b, c, do not change sign in D. In particular, a and c are never zero in D.

$$t_j = t_j(\epsilon t;\epsilon); \quad \frac{dt_{j+1}}{dt} = O\left(\epsilon\frac{dt_j}{dt}\right) \quad \text{uniformly in } D. \tag{3}$$

The last condition insures that each time scale is to be "slower" than the preceding one.

It is assumed that the multivariable expansions yield "generalized uniform asymptotic expansions" in the sense defined below.

DEFINITION. Let $\{S_j(t;\epsilon)\}$ be a sequence of functions defined for $t \in D$ and $0 \leqslant \epsilon \leqslant \epsilon_0$ for some $\epsilon_0 > 0$, and let $\{f_j(\epsilon)\}$, $\epsilon \longrightarrow 0$, be an asymptotic sequence. Then $\Sigma_{j=0}^M S_j(t;\epsilon)$ is a generalized uniform asymptotic expansion of

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AMS (MOS) subject classifications (1970). Primary 34E05, 34E10; Secondary 41A60.