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SIMPLE RIEMANN FUNCTIONS

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Research of Cohn [1] and Daggit [2] guesses or deduces functions M and G such that some Riemann functions have the form $R = Mf(G)$. The Riemann function is then found by solving an ordinary differential equation for f. Roughly speaking, all the results of Riemann, Cohn and Daggit are *simple* Riemann functions because (i) the equation is almost selfadjoint, (ii) the ordinary differential equation for f has coefficients depending on G, and (iii) the characteristic conditions for the Riemann function become initial conditions for f. These restrictions, formulated in Definition 2, seem mild because omitting any one of the three would allow every Riemann function to satisfy Definition 2.

Surprisingly, these conditions lead directly to expressions for M and G (Theorems 1 and 2). With these, *all* of Daggit's results are derived (Theorems 3 to 7) from *one* of his hypotheses. For a *selfadjoint* equation to have a simple Riemann function, Daggit's hypothesis is sufficient as well as necessary (Theorem 8). This announcement therefore contains proof that all *simple Riemann functions are known for selfadjoint equations*. The proofs for the general case are given in another paper [4].

DEFINITION 1. The unique *Riemann function R* for the equation

$$U_{xy} + A(x,y)U_x + B(x,y)U_y + C(x,y)U = 0 \tag{1.1}$$

satisfies the equation adjoint to (1.1), depends on two parameters X and Y, reduces to 1 at the point $(x,y) = (X, Y)$, and obeys

$$R_x = BR \text{ when } y = Y \quad \text{and} \quad R_y = AR \text{ when } x = X. \tag{1.2}$$

When a *multiplier M* is chosen, substituting $R = M(x,y,X,Y) \times U(x,y,X,Y)$ into the equation adjoint to (1.1) gives

$$U_{xy} + a(x,y,X,Y)U_x + b(x,y,X,Y)U_y + c(x,y,X,Y)U = 0. \tag{1.3}$$

If a *guess G* is then chosen, substituting $U = f(G(x,y,X,Y),X,Y)$ into (1.3) yields

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