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TRANSFERENCE RESULTS FOR MULTIPLIER OPERATORS

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The purpose of this paper is to show a transference result of the type obtained in [4] and [5] for convolution operators acting on functions defined on Σ_{n-1}, the unit sphere of $\mathbf{R}^n$. As a consequence we obtain a multiplier theorem for expansions in spherical harmonics and Gegenbauer polynomials. Also, Zygmund's inequality for Cesáro sums and that for Littlewood-Paley function g_δ, due to Bonami and Clerc [1], are easily obtained using our results [6]. I wish to express my appreciation to my Ph.D advisors, Professor R. Coifman and G. Weiss, for their encouragement and help in the preparation of this work.

Introduction. Let SO(n) be the group of all rotations of $\mathbf{R}^n$. The left regular representation of SO(n) defined by $R_u f(x) = f(u^{-1}x)$, $u \in \mathrm{SO}(n)$ and $f \in L^2(\Sigma_{n-1})$, decomposes into a direct sum of finite dimensional irreducible representations R^k ($n \geqslant 3$), $k = 0, 1, \ldots$. $L^2(\Sigma_{n-1}) = \Sigma_{k=0}^\infty H_k$, where H_k, the space of the representation R^k, consists of the spherical harmonics of degree k [2], [8], [9]. If $f \in L^2(\Sigma_{n-1})$, $f(x) = \Sigma_{k=0}^\infty (Z_{e,n-1}^{(k)} * f)(x)$, where $Z_{e,n-1}^{(k)}(x)$ is the zonal spherical harmonic of degree k and pole $e = (0, \ldots, 0, 1)$ and $*$ denotes convolution on Σ_{n-1}. A multiplier M, is an operator that commutes with the action of SO(n) on Σ_{n-1} and is defined on the class P of finite linear combinations of elements in the spaces H_k. Such M assume the form

$$Mf(x) = \sum m_k (Z_{e,n-1}^{(k)} * f)(x) \quad \text{(finite sum)}.$$

Multipliers for expansions in spherical harmonics. Let H be a Hilbert space over the complex numbers and let $L^p(\Sigma_{n-1}, H)$, $1 \leqslant p \leqslant \infty$, be the space of functions $f\colon \Sigma_{n-1} \longrightarrow H$ defined in the usual way replacing absolute values by $\|\cdot\|_H$. For the left regular representation of SO(n) on $L^2(\Sigma_{n-1}, H)$ we have a decomposition entirely similar to the one described above [3]. To a bounded operator on L^2 which commutes with rotations,corresponds a bounded sequence $\{m_k\}_{k=0}^\infty$ of operators on H such that $Mf(x) = \Sigma\, m_k (Z_{e,n-1}^{(k)} * f(x))$ (finite sum) for every $f \in P$. The operator valued function

$$K_r(x) = \sum_{k=0}^\infty r^k Z_{e,n-1}^{(k)}(x) m_k, \qquad r \in [0, 1),$$

is continuous. We write $Mf(x) = \lim_{r\to 1}(K_r * f)(x)$.

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