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ON SURFACES OBTAINED FROM QUATERNION ALGEBRAS OVER REAL QUADRATIC FIELDS[1]

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Let A be a totally indefinite division quaternion algebra with center $k = \mathbf{Q}(\sqrt{d})$, $d > 0$, $\mathcal{O}$ a maximal order in A, and $\Gamma(1) = \{\alpha \in \mathcal{O} \mid \nu(\alpha) = 1\}$ where ν is the reduced norm from A to k. Fix an isomorphism λ such that $A \otimes_Q \mathbf{R} \cong M_2(\mathbf{R}) \oplus M_2(\mathbf{R})$. Then $\lambda(\Gamma(1) \otimes_Q 1) \subseteq \mathrm{SL}_2(\mathbf{R}) \times \mathrm{SL}_2(\mathbf{R})$, and $j(\Gamma(1)) = \Gamma(1)/(\text{center } \Gamma(1))$ acts holomorphically and properly discontinuously on $X = H \times H$, where H is the usual upper half plane. In general, if Γ is any group of holomorphic automorphisms of X acting properly discontinuously and without fixed points, then $\Gamma\backslash X$ is a complex manifold. Since A is division the quotient is compact, and it is known to be a projective algebraic variety. In this note we discuss the numerical invariants and second cohomology group of $U(\Gamma) = \Gamma\backslash H \times H$ where Γ is commensurable with $\Gamma(1)$.

(A) For any algebraic number field F, a quaternion algebra with center F is determined up to isomorphism by a finite set $S(A)$ of prime divisors of F. Denote this algebra by $A(F, S(A))$.

THEOREM 1. *Assume $h(k)$ = class number of $k = 1$. Let $j(\Gamma(1)) = \Gamma(1)/\{\pm 1\}$, $A = A(k, S(A))$, and let*

$$\left(\frac{-}{p}\right)$$

be the Kronecker symbol. $j(\Gamma(1))$ acts on X without fixed points $\Leftrightarrow$ all of the following hold:

$$\left(\frac{-3}{p}\right) = 1 \quad or \quad \left(\frac{-D}{p}\right) = 1 \tag{1}$$

for some $P \in S(A)$, where $p\mathbf{Z} = P \cap \mathbf{Z}$ and $-D'$ is the discriminant of the field $\mathbf{Q}(\sqrt{-3d})$.

$$\left(\frac{-1}{p}\right) = 1 \ or \ \left(\frac{-D'}{p}\right) = 1 \tag{2}$$

for some $P \in S(A)$, where $p\mathbf{Z} = P \cap \mathbf{Z}$ and $-D'$ is the discriminant of the field $\mathbf{Q}(\sqrt{-d})$.

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[1] Partial results of the author's dissertation [3] under M. Kuga.