RESEARCH ANNOUNCEMENTS

EVERY PLANAR MAP IS FOUR COLORABLE[1]

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The following theorem is proved.

THEOREM. *Every planar map can be colored with at most four colors.*

As has become standard, the four color map problem will be considered
in the dual sense as the problem of whether the vertices of every planar graph
(without loops) can be colored with at most four colors in such a way that no
pair of vertices which lie on a common edge have the same color. The restriction
to triangulations with all vertices of degree at least five is a consequence of the
work of A. B. Kempe. Over the past 100 years, a number of authors including
A. B. Kempe, G. D. Birkhoff, and H. Heesch have developed a theory of
reducibility to attack the problem. Simultaneously, a theory of unavoidable
sets has been developed and the fusion of these has led to the proof.

A *configuration* is a subgraph of a planar triangulation consisting of a
circuit (called the ring) and its interior. A configuration is called *reducible* if it
can be shown by certain standard methods that it cannot be immersed in a
minimal counterexample to the four color conjecture. (For details, see [3] or
[4].) A set of configurations is called *unavoidable* if every planar triangulation
contains some member of the set. From the definitions, it is immediate that
the four color theorem is proved if an unavoidable set of reducible configurations
is provided.

The most efficient known method of producing unavoidable sets of con-
figurations is called the method of discharging. This method treats the planar
triangulation as an electrical network with charge assigned to the vertices.
Euler's formula is used to show that the initial charge distribution, giving
positive charge to vertices of degree five and negative charge to vertices of
degree greater than six, has positive total charge. Next, the initial charge is
redistributed in a manner which obeys the principle of conservation of charge.
This means that some vertices must end up with positive charge. Such an
algorithm can be made sufficiently sophisticated that a finite list of neighborhoods
of all possible vertices of ultimately positive charge can be described in detail.