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Markov chains, by D. Revuz, North-Holland Mathematical Library, vol. 11, North-Holland, Amsterdam; American Elsevier, New York, 1975, x + 336 pp., $35.50.

The theory of Markov processes can be considered in a great variety of settings. In the work under review the word "chain" is used to indicate discrete time (a convincing argument for this usage is made), the state space is a general measurable space, and the transition probabilities are assumed to be stationary. This context then determines the problems to be considered.

The most immediate problem, and historically the first to be pursued, concerns the asymptotic behavior of the n-step transition probabilities $P^n(x, A)$. In case the state space consists of a finite number of states only, this reduces to studying the asymptotic behavior of the nth power of a Markov matrix, and much early work was devoted to this situation. The case of general state space is of course much more complicated, and the pioneering work here is due to Doeblin. Between these two levels of generality lies that of denumerable state space, definitively treated by Kolmogorov, and alternatively by Feller.

In the ergodic theory of Markov chains one generally distinguishes between the recurrent and the transient case. Roughly, in the recurrent situation, a subset A of the state space will be visited infinitely often by the Markov chain started at x, for all (or most) starting points x, provided only that A is not too small (in a suitable sense). The parenthetical expressions can be made precise in various ways, leading to very different concepts of recurrence. In the denumerable case one may take "most" to mean all, and "small" to mean void. Following one of Doeblin's approaches for general state space, one can take "small" to mean of φ-measure zero, where φ is an auxiliary measure on the state space. Then taking "most" to mean all, one obtains the notion of φ-recurrence. A chain that is φ-recurrent for some φ is recurrent in the sense of Harris.

A subset A of the state space is closed if $P(x, A) = 1$ for all $x \in A$. No matter what notion of recurrence is used, the first problem is to show that the state space can be broken up into minimal closed sets, and the Markov chain restricted to any one of these sets is either recurrent or transient.

In the recurrent case one hopes to establish that $P^n(x, \cdot)$ is asymptotically independent of x (weak ergodicity); one can then expect convergence of