

$$\exp(xz + tz^2) = \sum_{n=0}^{\infty} v_n(x, t) z^n / n!;$$

other series of polynomials are also useful, just as series of polynomials other than  $\{z^n\}$  are important in complex analysis. Temperature functions possess a maximum principle, a reflection principle, and uniqueness theorems showing how they are determined by various kinds of data; there is even an analogue of Liouville's theorem. For a suitably restricted subclass of temperature functions there is Huygens' principle (which gets its name from a quite different analogous theory, optics, i.e. the theory of the wave equation); this says that the values of the function for some  $t$  can be used as initial data for determining the function at later values of  $t$ , in much the same way that we can take the values of an analytic function on a contour and use them in Cauchy's formula to calculate the function inside the contour. (Poisson's formula for harmonic functions is perhaps a closer analogue.) The analogy between positive temperature functions and positive harmonic functions has already been mentioned. One chapter is devoted to the use of Jacobian theta functions for solving the heat equation in a finite  $x$ -interval; the occurrence of these functions is less surprising than one might think, since the theta functions are series of functions  $k(x, t)$  or  $k_x(x, t)$ ; they also occur in the construction of the Green's function for an  $(x, t)$ -rectangle. One chapter indicates some possible generalizations to higher dimensions; another discusses homogeneous temperature functions ( $u(\lambda x, \lambda^2 t) = \lambda^n u(x, t)$ ). A final chapter considers several special topics.

The book is written in the author's customary polished but condensed style. Much of it consists of simplified versions of his own previous work. The results seem, generally speaking, to be more difficult than their analogues in complex analysis; I do not know whether this is because the latter theory is longer established or because problems about the heat equation are inherently more difficult than problems for Laplace's equation (as suggested to me by A. Friedman). It seems likely, however, that many additional interesting results are waiting to be discovered (or invented, depending on our philosophy of mathematics). Anyone who wishes to participate in the search should have this book at hand.

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*Continuous flows in the plane*, by Anatole Beck, Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen, Band 201, Springer-Verlag, New York, Heidelberg, Berlin, 1974, x + 462 pp., \$46.80.

A *flow* in a space  $X$  is a (continuous) group action of the real line on  $X$ ; that is, a continuous function  $\varphi: \mathbf{R} \times X \rightarrow X$  such that  $\varphi(t + s, x) = \varphi(t, \varphi(s, x))$ . Behind this simple analytic veil lies, in the case where  $X$  is the plane  $\mathbf{R}^2$  (or the two-sphere  $S^2$ ), a beautiful geometric theory. The plane becomes a patchwork quilt. The patches come in infinitely varied and intriguing patterns, that, nevertheless, admit to a surprising amount of classification.