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SINGULAR HAMMERSTEIN EQUATIONS AND MAXIMAL MONOTONE OPERATORS

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Consider the nonlinear integral equation of Hammerstein type

$$u(x) + \int_\Omega k(x, y)f(y, u(y))\beta(dy) = h(x) \qquad (x \in \Omega), \tag{1}$$

where h and the solution u lie in a space X of measurable functions on Ω. The Hammerstein equation is said to be regular if for

$$F(u)(y) = f(y, u(y)) \qquad (y \in \Omega); \qquad Kv(x) = \int_\Omega k(x, y)v(y)\beta(dy) \qquad (x \in \Omega), \tag{2}$$

the operator KF is defined on all of X, and singular otherwise.

In some recent papers (summarized in [2]), the writers have studied the existence theory for regular Hammerstein equations in $L^p(\beta)$ with $1 < p \leqslant +\infty$ under very general assumptions on K and F. In later papers (cf. [4]), one of the writers has obtained general existence results for the singular case, using measure-theoretic arguments and mild compactness assumptions on K. We present results here without compactness assumptions based on a new theorem on linear monotone operators.

THEOREM 1. *Let X be a reflexive Banach space, L_0 and L_1 linear monotone mappings from X into 2^{X^*} with $L_0 \subseteq L_1^*$. Then there exists a maximal monotone linear map from X into 2^{X^*} such that $L_0 \subseteq L \subseteq L_1^*$.*

For single-valued, densely defined maps in Hilbert space, this coincides with a theorem of R. S. Phillips [6] obtained using ideas of M. Kreĭn [5]. For reflexive Banach spaces, in general, we have as a corollary a result obtained in 1968 by one of the writers [1]:

THEOREM 2. *Let X be a reflexive Banach space, L a closed linear monotone map from X into 2^{X^*}. Then L is maximal monotone if and only if L^* is monotone.*

We sketch the proof of Theorem 1 (detailed proofs are given in [3]). By a Zorn's Lemma argument we may construct a monotone linear map L with $L_0 \subseteq L \subseteq L_1^*$ such that L is maximal monotone *in the graph of* L_1^*. Let J be a duality map of X into X^* corresponding to a norm on X with X and X^* locally uniformly convex.

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