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# ELLIPTIC PSEUDO DIFFERENTIAL OPERATORS DEGENERATE ON A SYMPLECTIC SUBMANIFOLD

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1. **Introduction.** This note is concerned with the classes of pseudo differential operators $L^{m,M}(\Omega, \Sigma)$, $\Sigma$ symplectic submanifold of codimension 2, in Sjöstrand **[4]**; the definitions of $P$ in $L^{m,M}(\Omega, \Sigma)$ and of the associated winding number $N$ are recalled in §2. In Helffer **[2]** the study of the hypoellipticity of $P$ is reduced to the analysis of the bounded solutions of an ordinary differential equation. Here we deduce an explicit result for $N = 2 - M$: essentially, we can prove that in this case all the bounded solutions are products of an exponential function with polynomials.

2. **The classes $L^{m,M}(\Omega, \Sigma)$ and the winding number.** Let $\Omega \subset \mathbf{R}^n$ be an open set. Let $\Sigma \subset T^*(\Omega)\backslash 0$ be a closed conic symplectic submanifold of codimension 2 ($\Sigma$ symplectic means that the restriction of the symplectic form $\omega = \Sigma d\xi_s \wedge dx_s$ to $\Sigma$ is nondegenerate). $L^{m,M}(\Omega, \Sigma)$ is the set of all the pseudo differential operators $P$ which have a symbol of the form

$$p(x, \xi) \sim \sum_{j=0}^{\infty} p_{m-j/2}(x, \xi), \tag{1}$$

where $p_{m-j/2}$ is positively homogeneous of degree $m - j/2$ and for every $K \subset\subset \Omega$ there exists a constant $C_K$ such that

$$|p_m(x, \xi)|/|\xi|^m \geqslant C_K^{-1} d_\Sigma^M(x, \xi), \tag{2}$$

$$|p_{m-j/2}(x, \xi)|/|\xi|^{m-j/2} \leqslant C_K d_\Sigma^{M-j}(x, \xi), \qquad 0 \leqslant j \leqslant M, \tag{3}$$

for all $(x, \xi) \in K \times \mathbf{R}^n$, $|\xi| > 1$ ($d_\Sigma(x, \xi)$ is the distance from $(x, \xi/|\xi|)$ to $\Sigma$).

Fix $\rho$ in $\Sigma$, denote by $N_\rho(\Sigma)$ the orthogonal space of $T_\rho(\Sigma)$ with respect to $\omega$ and choose two linear coordinates on $N_\rho(\Sigma)$ $u_1$, $u_2$ such that $\omega/N_\rho(\Sigma) = du_2 \wedge du_1$. Take $X = (u_1, u_2) \in N_\rho(\Sigma)$ and let $V$ be any vector field on $T^*(\Omega)$ equal to $X$ at $\rho$. We define the homogeneous polynomial

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