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# INFINITE DIMENSIONAL VERSIONS OF TWO THEOREMS OF CARL SIEGEL[1]

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1. **The center theorem.** Let $U$ be an open neighborhood of the origin in a Banach space $E$ and $f$: $U \longrightarrow E$ an analytic function with constant term $f(0) = 0$ and linear term $u = (Df)_0$ a toplinear isomorphism.

In 1871 Ernst Schröder [4], motivated by problems in the iteration of functions, considered the problem of when $f$ is analytically conjugate to its linear part $u$, that is, when does there exist an analytic function $\varphi$ with $\varphi(0) = 0$ and $(D\varphi)_0 = \mathrm{id}_E$ such that $\varphi^{\leftarrow} \circ f \circ \varphi = u$. Jules Farkas [2] and Gabriel Koenigs [3] independently settled this question for contractions, $|u| < 1$, in one complex dimension by showing in 1884 that there always exists an analytic Schröder series.

The center case, $|u| = 1$, remained an enigma for fifty-eight years after the work of Farkas and Koenigs, although many people attacked the problem in the 'twenties' and 'thirties'. Let

$$(\mathrm{id} - u_*^{-1}u^*)_k\colon \mathcal{P}_k(E, E) \longrightarrow \mathcal{P}_k(E, E)$$

be defined on $k$-homogeneous polynomials $p$ on $E$ with coefficients in $E$ by

$$(\mathrm{id} - u_*^{-1}u^*)_k\colon p \longmapsto p - u^{-1} \cdot p \circ u.$$

In 1942 Carl Siegel [5] introduced his multiplicative small divisor diophantine inequalities: say that $u$ is "non-Liouville" iff there are constants $c, \nu$ such that, for each $k > 2$, $(\mathrm{id} - u_*^{-1}u^*)_k$ is an isomorphism and $|(\mathrm{id} - u_*^{-1}u^*)_k^{-1}| < ck^{\nu}$. In his celebrated six page paper Siegel proved in the one dimensional complex case the Siegel center theorem: *If u is non-Liouville then f is analytically conjugate to u.* The proof is by the majorant method and depends on delicate number-theoretical estimates. Shlomo Sternberg [7] extended the center theorem to the finite dimensional case in 1961.

Meanwhile several people, including John Nash, Andrei Kolmogorov, Vladimir Arnol'd, and Jürgen Moser, had developed a general technique for handling

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[1]Most of the results announced comprise part of a thesis written at the University of California, Santa Cruz under the direction of Al Kelley.