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# TWO TORSION IN $H$-SPACES

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The objective of this note is to announce theorems about two torsion in $H$-spaces. We say that $X$ is a *finite* $H$-space if it has the homotopy type of a finite CW complex. Throughout this paper we assume that $X$ is any simply connected, finite $H$-space whose mod two homology ring $H_*(X; Z_2)$ is associative. Every known finite simply connected $H$-space satisfies this assumption.

THEOREM 1. *Let $\Omega X$ be the space of basepointed loops on $X$. Then $H_*(\Omega X; Z)$ has no two torsion, and $H_*(\Omega X; Z_2)$ is concentrated in even degrees.*

THEOREM 2. *The two torsion coefficients of $H^*(X; Z)$ are of order at most two.*

THEOREM 3. *There are no even degree algebra generators in the* mod *two cohomology ring, $H^*(X; Z_2)$.*

THEOREM 4 (HUREWICZ MAP). *The kernel of the two-local Hurewicz homomorphism $h_* \otimes Z_{(2)}\colon \Pi_*(X) \otimes Z_{(2)} \longrightarrow H_*(X; Z_{(2)})$ is the two torsion of $\Pi_*(X)$.*

Thus, the first nonvanishing homotopy group of $X$ is two torsion free.

In the case that $X$ is a simply connected Lie group, the above results were proven by appealing to the underlying differentiable manifold structure of the group. Details and proofs will appear elsewhere.

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