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A CHARACTERIZATION OF THE SUBGROUPS OF FINITELY PRESENTED GROUPS[1]

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ABSTRACT. We characterize the subgroups of finitely presented groups as those countable groups having consistently recursively enumerable word problem. We also characterize the subgroups of simple subgroups of finitely presented groups as those countable groups having consistently recursive word problem.

Graham Higman has characterized the finitely generated subgroups of finitely presented groups as those finitely generated groups with recursively enumerable (r.e.) word problem (see [H]). In the same paper he asked for a characterization of *all* countable groups which are subgroups of finitely presented groups. Theorem I below gives such a characterization.

DEFINITION 1. A countable group G has *r.e. word problem relative to a set* S of generators if and only if there is a partial algorithm which converges on all words on S which equal 1 in G and gives answer "yes", and does not give answer "yes" on any word $\neq 1$ in G. (It could not converge, or it could converge and answer "no".) If the word problem for G is r.e. relative to some generating set S, we will say that G has *r.e. word problem.*

DEFINITION 2. G *has recursive word problem relative to* S if there is a partial algorithm which converges for all words w on S, and answers correctly whether or not $w = 1$ in G. If the word problem for G is recursive relative to some generating set, we will say G has *solvable word problem.*

DEFINITION 3. If A is a recursive alphabet and if f is a partial algorithm from the set of words on A to the set "yes", "no" we call f a *consistent algorithm* if the set of conditions $\{W = 1 \mid f(W) = \text{"yes"}\} \cup \{W \neq 1 \mid f(W) \neq \text{"yes"}\}$ is consistent with the axioms of group theory.

If f is a consistent algorithm, then the set of words W such that $f(W) =$ "yes" is closed under concatenation, and conjugation by arbitrary words U on A. In group theoretic terms this means that $f(U) =$ "yes" if and only if $f(U') =$ "yes" where U' is the freely reduced word equivalent to U, and if F is the free group on A, the set of equivalence classes of words W such that $f(W) =$ "yes" is a normal subgroup of F.

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