

THE ROOTS OF A SIMPLE LIE ALGEBRA ARE LINEAR

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In this note we announce a result on the structure of a Cartan subalgebra of a finite-dimensional simple Lie algebra over an algebraically closed field F of characteristic $p > 7$. Consequences of this result include the linearity of the roots of a finite-dimensional simple Lie algebra over F and classification of the finite-dimensional simple Lie algebras over F whose roots generate a cyclic group.

1. **Structure of \bar{H} .** Let F be an algebraically closed field of characteristic $p > 7$. Let L be a finite-dimensional simple Lie algebra over F and let H be a Cartan subalgebra of L . Identify L with the subalgebra $\text{ad } L$ of the restricted Lie algebra $\text{Der } L$. Let \bar{H} denote the restricted subalgebra of $\text{Der } L$ generated by H . Since \bar{H} is restricted, each element of \bar{H} has a Jordan-Chevalley decomposition into its semisimple and nilpotent parts ([3, §1] or [4, Chapter V.7]).

Now \bar{H} is a nilpotent restricted Lie algebra and hence contains a unique maximal torus T . Also \bar{H} contains a nil ideal I , called the nil radical, which contains every nil ideal. (An ideal is nil if every element in it is nilpotent.)

Our main result is

THEOREM 1. $\bar{H} = T + I$.

Since every root in the Cartan decomposition for L extends to a function on \bar{H} which is linear on $T + I$ we have

COROLLARY. *Every root in the Cartan decomposition for L (with respect to H) is a linear function on H .*

The first result of this type is due to Schue [3], who proved $\bar{H} = T + I$ under the hypotheses that $\dim T = 1$ and that every proper subalgebra of L is solvable.

To prove Theorem 1 (in [6]) we assume $\bar{H} \neq T + I$. Then (following Schue) we find an element $b \in \bar{H}$ such that $b^p \in I$, $[b, \bar{H}] \subseteq T + I$, and $[b, \bar{H}] \not\subseteq I$. We let $L = H + \sum_{\gamma \in \Gamma} L_{\gamma}$ be the Cartan decomposition of L with respect to H and set

$$S = \{(\gamma, \delta) \in \Gamma \times \Gamma \mid \gamma([b[L_{\delta}, L_{-\delta}]]) \neq (0)\}.$$

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