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THE EXISTENCE OF NONTRIANGULABLE CUT LOCI

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We continue in this note the description of deformation theorems for geodesic fields on a Riemannian manifold begun in [1], restricting ourselves here to surfaces of revolution and to deformations of metric within this class. Using real and holomorphic Fourier transforms, we obtain in Theorem 2 an explicit formula for the deformation of metric corresponding to a prescribed deflection of geodesics. As an application, we turn again to the structure of the cut locus and prove

THEOREM 1. *There exists in R^3 a strictly convex surface of revolution containing a nonempty open set of points p for which the cut locus $C(p)$ is nontriangulable.*

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1. **Geodesics on a surface of revolution.** Let M be a surface of revolution whose metric is given in polar coordinates on the disc $r \leqslant 1$ by

$$ds^2 = E(r)dr^2 + r^2\,d\theta^2.$$

Then the equation of a geodesic $\gamma(t) = (r(t), \theta(t))$ is given explicitly [4] by

$$\theta = \theta_0 + \int_{r_0}^{r} \frac{c\sqrt{E(r)}}{r\sqrt{r^2 - c^2}}\,dr \tag{1}$$

where the quantity $|c|$ measures the closest approach (in the r-θ plane) of the geodesic to the origin. Note that the constant c can be computed from any small segment of the geodesic by Clairaut's theorem [4]:

$$c = r(t)\sin \epsilon(t), \tag{2}$$

where $\epsilon(t)$ is the angle between γ and the meridian θ = constant.

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