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INDECOMPOSABLE MODULES: MODULES WITH CORES

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We introduce a class of modules, called modules with cores, larger than any previously known fundamental class of indecomposable modules. The core of a module M is the intersection, $C(M)$, of its nonsuperfluous submodules; that is, the intersection of submodules N such that $N + N' = M$ for some proper submodule N'. Dually the cocore, $C^0(M)$, is the sum of the nonessential submodules of M. Any module with a core (meaning $C(M) \neq 0$) or a cocore ($C^0(M) \neq M$) is indecomposable.

Our main aim is to describe a classification of radical squared zero Artin algebras such that every indecomposable module of finite length has a core or a cocore. (Finite dimensional algebras over a field are the most important examples of Artin algebras.) The classification shows that there are modules having a core and no cocore. This establishes our claim concerning the size of the class of modules with cores even in the restricted setting of radical squared zero Artin algebras of finite representation type—for modules with waists (see [1]) have a core and a cocore and include, as well as serial modules, modules with simple socles or, dually, simple tops.

The proof of the following existence theorem is constructive and uses a result announced in [5] concerning indecomposability of amalgamations of basic modules of finite length. (A nonzero module is basic if it is its own core. For modules of finite length this just means that the module has a simple top.)

THEOREM. *If a left Artin ring has a basic module containing three nonisomorphic simple submodules then it has an indecomposable module of finite length with neither a core nor a cocore.* □

It is an open question whether the theorem is valid without the assumption that the specified simple modules are nonisomorphic. Indeed the assumption is unnecessary when the underlying ring is an Artin algebra with square 0 radical. This fact, which can be verified using techniques of Auslander and Reiten [2], plays a role in the proof of our main result. We denote the length of a module

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