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SPECTRAL CLASSIFICATION OF OPERATORS AND OPERATOR FUNCTIONS

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Let A and B be holomorphic functions from an open set Ω in the complex plane into the Banach space $L(X, Y)$ of all bounded linear operators between two Banach spaces X and Y. The functions A and B are called *equivalent on* Ω (see [3]) if there exist holomorphic operator functions E and F on Ω, whose values are bijective bounded linear operators on X and Y, respectively, such that

$$A(\lambda) = F(\lambda)B(\lambda)E(\lambda), \quad \lambda \in \Omega.$$

The concept of equivalence is also of interest in the case that ($X = Y$ and) A and B are linear functions of the form $C - \lambda I$. In that case it provides a language in which the spectral structure of a linear operator at a point may be classified. Two operators T and S are said to have the same *spectral structure* at a point λ_0 if the operator functions $T - \lambda I$ and $S - \lambda I$ are equivalent on an open neighbourhood of λ_0. More generally, we say that two operator functions A and B belong to the same *spectral class* at a point λ_0 if there exists an open neighbourhood of λ_0 on which A and B are equivalent.

Let $D(\lambda) = (\lambda - \lambda_0)^{k_1}P_1 + \cdots + (\lambda - \lambda_0)^{k_n}P_n + P_0$, where $k_1, \ldots, k_n$ are positive integers, $P_1, \ldots, P_n$ are mutually disjoint one dimensional projections and $P_0 = I - (P_1 + \cdots + P_n)$. An operator function A belongs to the spectral class generated by D at λ_0 if and only if $A(\lambda)$ is bijective for λ near λ_0, $A(\lambda_0)$ is Fredholm and the partial multiplicities of A at λ_0 are given by the numbers $k_1, \ldots, k_n$ (see [5]). Other examples of spectral classes can be found in [1] and [7].

The problem of finding the simplest representative of a spectral class containing a given operator function A has many variants. One possibility is to look for functions of the form $T - \lambda I$. It is clear that in this way the problem is not always solvable. However after a suitable extension of the operator function A and the underlying spaces the problem has a positive solution.

If Z is a Banach space, the *Z-extension* of A is the operator function whose value at λ is the operator $A(\lambda) \oplus I_Z$ in $L(X \oplus Z, Y \oplus Z)$, i.e., the direct sum of

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