BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 4, July 1976

GLOBAL BIFURCATION THEOREMS FOR NONLINEARLY PERTURBED OPERATOR EQUATIONS

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Communicated by R. K. Miller, February 3, 1976

1. **Introduction.** The author [2], [3], and [4] has previously studied the equation

$$Lu = \lambda u + H(\lambda, u) \tag{1}$$

in a real Banach space B where L is linear and H is compact and $o(\|u\|)$ is uniformly on bounded λ intervals. In that setting, if λ_0 is an isolated normal eigenvalue of L having odd algebraic multiplicity, then $(\lambda_0, 0) \in R \times B$ is a bifurcation point for (1). Moreover, a continuous branch of solutions emanates from each of these points and obeys a threefold alternative.

This paper combines methods of the author and Stuart [7] to show that similar results hold if $H(\lambda, u)$ is merely continuous and $o(\|u\|)$ uniformly on bounded λ intervals.

2. **Preliminaries.** In this paper all work is a real Banach space B. L denotes a linear operator densely defined in B, and H represents a continuous operator that is $o(\|u\|)$ near $u = 0$ uniformly on bounded λ intervals. Define the essential spectrum of L as the members of the spectrum of L which are not isolated normal eigenvalues of L and denote this set by $e(L)$.

We consider a normal eigenvalue λ_0 of L. Let

$$\alpha_{\lambda_0} = \sup \{\gamma \mid \gamma \in e(L), \gamma < \lambda_0\} \quad \text{and} \quad \beta_{\lambda_0} = \inf \{\gamma \mid \gamma \in e(L), \gamma > \lambda_0\}$$

respectively if the corresponding sup or inf exists. Otherwise, set $\alpha_{\lambda_0} = -\infty$ and/or $\beta_{\lambda_0} = +\infty$. Assume for now that α_{λ_0} and β_{λ_0} are both finite. For $\epsilon > 0$, the only members of the spectrum of L in $(\alpha_{\lambda_0} + \epsilon, \beta_{\lambda_0} - \epsilon)$ are normal eigenvalues of L. If P_ϵ denotes the projector onto the direct sum of the eigenspaces of these eigenvalues and $Q_\epsilon = I - P_\epsilon$, then it has been shown [2], [3] and [4] that

$$u = \frac{(L - \mu_0)P_\epsilon u}{\lambda - \mu_0} + \left((L - \lambda)^{-1}Q_\epsilon - \frac{P_\epsilon}{\lambda - \mu_0}\right) H(\lambda, u) \tag{2}$$

is equivalent to (1) for λ in $[\alpha_{\lambda_0} + \epsilon, \beta_{\lambda_0} - \epsilon]$ and μ_0 any member of the resolvent of L not lying in $(\alpha_{\lambda_0}, \beta_{\lambda_0})$ ($(L - \lambda)^{-1}$ is defined on $Q_\epsilon B$).

AMS (MOS) subject classifications (1970). Primary 46N05.