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RANDOM REARRANGEMENTS OF FOURIER COEFFICIENTS

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In [2] and [3] Hardy and Littlewood characterized those sequences of numbers which for every variation of their arguments and arrangements are the Fourier coefficients of a function in L^p $(2 < p < \infty)$ and those sequences which for some variation of their arguments and arrangements are the Fourier coefficients of a function in L^p $(1 < p < 2)$. In [4] Paley and Zygmund characterized those sequences (c_n) such that for almost every choice of ± 1's,$(\pm c_n)$ is the sequence of Fourier coefficients of a function in L^p $(1 \leqslant p < \infty)$. Here we are interested in the following problem: Which sequences are for almost every variation of their arrangements the sequence of Fourier coefficients of a function in L^p? Of course it is necessary to make precise the phrase "almost every variation of their arrangements". Following Garsia [1] we consider a probability space X of "local" permutations of $\{1, 2, \ldots\}$: For $k = 0, 1, 2, \ldots$ let $S(2^k)$ be the symmetric group on the set $\{2^k, 2^k + 1, \ldots, 2^{k+1} - 1\}$. To each $\sigma_k \in S(2^k)$ we assign the probability $1/2^k$, and we let X be the product probability space $\Pi_{k=0}^{\infty} S(2^k)$. For $\sigma = (\sigma_0, \sigma_1, \ldots) \in X$ and a Fourier series of the form $\Sigma_{n=1}^{\infty} c_n e^{in\theta}$, we define

$$S(\sigma, \theta) = \sum_{k=0}^{\infty} \sum_{2^k \leqslant n < 2^{k+1}} c_{\sigma_k(n)} e^{in\theta}.$$

For $k = 0, 1, \ldots$ let

$$a_k = 2^{-k} \sum_{2^k \leqslant n < 2^{k+1}} c_n,$$

and let $(*)$ stand for the statement that

$$\sum_{k=0}^{\infty} |a_k|^p 2^{k(p-1)} < \infty \quad \text{and} \quad \sum_{k=0}^{\infty} \sum_{2^k \leqslant n < 2^{k+1}} |c_n - a_k|^2 < \infty.$$

THEOREM 1. *For $2 < p < \infty$, the following are equivalent*:

(i) *for some $\sigma \in X$, $S(\sigma, \theta)$ is an L^p Fourier series*;

(ii) *$S(\sigma, \theta)$ is almost surely an L^p Fourier series*;

(iii) $(*)$.

THEOREM 2. *For $1 < p < 2$, the following are equivalent*:

(i) *$S(\sigma, \theta)$ is an L^p Fourier series with positive probability*;

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