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Commutative formal groups, by Michel Lazard, Lecture Notes in Mathematics, vol. 443, Springer-Verlag, Berlin, Heidelberg, New York, 1975, 236 pp., $9.90.

This is the book we have been waiting for ever since P. Cartier's pair of notes in the Comptes Rendus of 1967. In these, Cartier sketched a thoroughgoing extension of the Dieudonné theory that had already classified commutative formal groups over a perfect field of characteristic p, in terms of modules over a certain noncommutative ring. But Cartier left the job of exposition unfinished, and Lazard has done us the service of organizing the material, filling in all the details, and adding a quantity of his own results, so that we finally have a basic reference on this aspect, probably the central aspect, of the theory of commutative formal groups.

An n-dimensional (coordinatized) formal group is simply an n-tuple $\mathbf{F} = (F_1, \ldots, F_n)$ of formal power series, subject to a single condition expressing a kind of associativity. Here, $F_i = F_i(\mathbf{x}, \mathbf{y})$, $\mathbf{x} = (x_1, \ldots, x_n)$, $\mathbf{y} = (y_1, \ldots, y_n)$; and $x_1, \ldots, y_n$ are $2n$ independent indeterminates. For instance, the expansion at the origin of the group law of an n-dimensional complex analytic Lie group gives rise to such series, once a coordinate system is chosen; the standard coordinatization of the one-dimensional multiplicative Lie group $\mathbf{C}^*$, for example, gives the single power series $F(x,y) = x + y + xy$.

The advantage in talking about formal groups rather than local groups is that the single relation of associativity $\mathbf{F}(\mathbf{F}(\mathbf{x}, \mathbf{y}), \mathbf{z}) = \mathbf{F}(\mathbf{x}, \mathbf{F}(\mathbf{y}, \mathbf{z}))$ makes sense algebraically, in the ring of formal power series $A[[\mathbf{x}, \mathbf{y}, \mathbf{z}]]$, where A is any commutative ring whatever. We need not restrict ourselves to the groundrings $\mathbf{C}$ and $\mathbf{R}$, not even to topological rings, and can now ask the relationship between Lie algebras over the ring A and formal groups over A. We then find that if A is a $\mathbf{Q}$-algebra, i.e. if every positive integer is invertible in A, then the categories of finite-dimensional Lie algebras over A and of