BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 3, May 1976

INFINITE LOOP MAPS AND THE COMPLEX J-HOMOMORPHISM

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Communicated by P. T. Church, December 23, 1975

ABSTRACT. We study the complex J-homomorphism $j: U \to SG$ as the composition of two infinite loop maps.

1. **Introduction.** Let p be an odd prime and let q be a prime generating the units of Z/p^2. All spaces will be p-localized. The solution of the Adams conjecture establishes a commutative diagram of fibre sequences.

$$\begin{array}{ccccccccccc}
\cdots \longrightarrow & U & \xrightarrow{\psi^q - 1} & U & \xrightarrow{\omega} & J^{\oplus} & \longrightarrow & BU^{\oplus} & \xrightarrow{\psi^q - 1} & BU^{\oplus} \\
 & & & \| & & \downarrow{\scriptstyle \mu} & & \downarrow{\scriptstyle \tau} & & \| \\
\cdots\cdots \longrightarrow & & & U & \xrightarrow{j} & SG & \longrightarrow & SG/U & \longrightarrow & BU^{\oplus}
\end{array} \tag{1.1}$$

Several, possibly different, τ have been constructed ([2], [5] and [8]). Given τ, then μ is unique. The fibre sequences are sequences of infinite loop maps and it is natural to ask whether (1.1) can be extended arbitrarily to the right—the infinite loop Adams conjecture. By [4] this would be true if τ were an infinite loop map. These results suggest strongly the validity of the conjecture.

In [2] an H-map, τ, is given. If $\mathbf{F}_q$ is the field with q elements the finite dimensional vector spaces over $\mathbf{F}_q$ under direct sum form a permutative category from which the infinite loopspace $J^{\oplus}$ is constructed by the technique of [1]. Similarly SG is obtained from a category of finite sets under cartesian product. The forgetful functor gives the "discrete models" infinite loop maps $\delta: J^{\oplus} \longrightarrow SG$.

THEOREM 1. *If τ is the map constructed in* [2] *then $\mu = \delta$ in* (1.1).

$J^{\otimes}$ is the infinite loopspace obtained from a category of vector spaces of $\mathbf{F}_q$ under tensor product. Assigning to a set the vector space generated by its elements gives $\nu: SG \longrightarrow J^{\otimes}$. Define Coker $J^{\otimes}$ by the infinite loop fibering Coker $J^{\otimes} \xrightarrow{\pi} SG \xrightarrow{\nu} J^{\otimes}$.

THEOREM 2. *$\nu \circ f: J^{\oplus} \longrightarrow J^{\otimes}$ is a homotopy equivalence for any map $f: J^{\oplus} \longrightarrow SG$ such that $f_{\#}$ is nontrivial on π_{2p-3}.*

AMS (MOS) subject classifications (1970). Primary 55E50, 55F25; Secondary 55D35, 55B20.

Key words and phrases. J-homomorphism, infinite loop Adams conjecture, transfer, permutative category, discrete models map.