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MAYER-VIETORIS SEQUENCES FOR COMPLEXES OF DIFFERENTIAL OPERATORS

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This is an announcement of some of the results in [1].

1. **Preliminaries.** Let X be a smooth manifold, E^i, $i = 0, 1, \ldots$, smooth vector bundles, and $\Omega \subset X$ open. Let $\mathcal{E}^i(\Omega) = C^\infty(\Omega, E^i)$. We consider complexes of linear differential operators with locally constant orders

$$\mathcal{E}(\Omega)\colon\ \mathcal{E}^0(\Omega) \xrightarrow{D^0} \mathcal{E}^1(\Omega) \xrightarrow{D^1} \cdots.$$

The cohomology of $\mathcal{E}(\Omega)$ is $H^i(\Omega) = \ker D^i/\mathrm{im}\, D^{i-1}$. Let $S \subset \Omega$ be a smooth hypersurface dividing Ω into two parts: $\Omega - S = \mathring{\Omega}^+ \cup \mathring{\Omega}^-$; $\mathring{\Omega}^+ \cap \mathring{\Omega}^- = \varnothing$; and $S \cup \mathring{\Omega}^\pm = \Omega^\pm$. Let $\mathcal{E}^i(\Omega^\pm)$ be the sections over $\Omega^\pm$ smooth up to S. We obtain

$$\mathcal{E}(\Omega^\pm)\colon\ \mathcal{E}^0(\Omega^\pm) \xrightarrow{D^0} \mathcal{E}^1(\Omega^\pm) \xrightarrow{D^1} \cdots.$$

A section $u \in \mathcal{E}^i(\Omega)$ has zero Cauchy data on S if $D^i\tilde{u} = \tilde{f}$ is valid on Ω in the sense of distributions where $\tilde{u} = u$ on Ω^+ and $= 0$ on $\Omega - \Omega^+$, and $\tilde{f} = D^i u$ on Ω^+ and $= 0$ on $\Omega - \Omega^+$; and similarly with Ω^+ replaced by Ω^-. The space of such sections is $\mathcal{I}(\Omega, S)$, and $\mathcal{I}(\Omega^\pm, S) = \mathcal{I}(\Omega, S)|_{\Omega^\pm}$. We obtain complexes

$$\mathcal{I}(\Omega, S)\colon\ \mathcal{I}^0(\Omega, S) \xrightarrow{D^0} \mathcal{I}^1(\Omega, S) \xrightarrow{D^1} \cdots,$$

and

$$\mathcal{I}(\Omega^\pm, S)\colon\ \mathcal{I}^0(\Omega^\pm, S) \xrightarrow{D^0} \mathcal{I}^1(\Omega^\pm, S) \xrightarrow{D^1} \cdots,$$

with cohomologies $H^i(\Omega, \mathcal{I})$ and $H^i(\Omega^\pm, \mathcal{I})$, respectively.

The tangential complex is the quotient complex $0 \longrightarrow \mathcal{I}(\Omega, S) \longrightarrow \mathcal{E}(\Omega) \longrightarrow \mathcal{C}(S) \longrightarrow 0$. An element of $\mathcal{C}^i(S)$ is Cauchy data for D^i, the induced operator is D^i_S, and the cohomology is $H^i(S)$.

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