

INTRINSIC DISTANCES, MEASURES AND GEOMETRIC FUNCTION THEORY¹

BY SHOSHICHI KOBAYASHI

To the memory of Carl B. Allendoerfer

Table of Contents

| | |
|--|-----|
| 1. Introduction | 357 |
| 2. Intrinsic pseudo-distances | 359 |
| 3. Hyperbolic complex spaces | 364 |
| 4. Metric and holomorphic completeness | 367 |
| 5. Intrinsic volume forms | 369 |
| 6. Schwarz lemma and differential geometric criteria for hyperbolicity | 371 |
| 7. Ample canonical bundles and manifolds of general type | 374 |
| 8. Families of holomorphic and meromorphic maps | 377 |
| 9. Automorphisms and endomorphisms of complex spaces | 383 |
| 10. Extension of holomorphic and meromorphic maps | 386 |
| 11. Defect relations | 387 |
| 12. Examples and applications | 390 |
| 13. Unsolved problems | 399 |

1. **Introduction.** In 1879, Picard discovered the following spectacular theorems:

THE LITTLE PICARD THEOREM. *If a function $f(z)$ holomorphic in $|z| < \infty$ misses two values, say 0 and 1, then $f(z)$ is constant.*

THE GREAT PICARD THEOREM. *If a function $f(z)$ holomorphic in $0 < |z| < r$ misses two values, say 0 and 1, then $z=0$ is either a removable singularity or a pole.*

The original proofs of these theorems involved the use of the modular function $\lambda: H \rightarrow \mathbf{C} - \{0, 1\}$, where H denotes the upper half-plane $\{z = x + iy; y > 0\}$. Subsequent successful efforts by E. Borel, Landau, Schottky, Montel, Bloch and others to find so-called "elementary" proofs (i.e., proofs free of the modular function) greatly enriched the theory of functions.

In his epoch making paper of 1925, R. Nevanlinna established the theory of value distributions. The so-called defect relation for a meromorphic

An expanded version of an invited address delivered before the Annual Meeting of the Society in San Francisco on January 15, 1974; received July 16, 1975.

AMS (MOS) subject classifications (1970). Primary 32H15, 32H20, 32H25; Secondary 32A15, 32A17, 32C10, 32E10, 32F15, 32H10, 32J05, 32J25, 32M05, 30A70, 14E05, 53C55.

¹ Partially supported by NSF Grant GP-42020X.