BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 2, March 1976

GENERATORS OF THE UNITARY Z/p BORDISM RING

BY CZES KOSNIOWSKI

Communicated by P. T. Church, December 29, 1975

I. **Serendipity**. Let p be a prime number, let $\mathbf{Z}/p$ be the cyclic group of order p and let $\mathfrak{A}_*^{\mathbf{Z}/p}$ be the unitary $\mathbf{Z}/p$ bordism ring.

THEOREM 1. *$\mathfrak{A}_*^{\mathbf{Z}/p}$ is multiplicatively generated, over $\mathfrak{A}_*$, by the following*:

$$\{\Gamma^m(\mathrm{pt}), m \geqslant 0\},$$
$$\bigcup\{\mathbf{Z}/p\},$$
$$\bigcup\{\Gamma^m(\mathbf{C}P_j^1), m \geqslant 0, (p+1)/2 \leqslant j < p-1\},$$
$$\bigcup\{S_j, 1 \leqslant j \leqslant (p-1)/2\},$$
$$\bigcup\{\Gamma^m(C_j), m \geqslant 0, 1 < j \leqslant (p+1)/2\},$$
$$\bigcup\{\Gamma^m(\mathbf{C}P_j^n), m \geqslant 0, n \geqslant 2, 1 \leqslant j \leqslant p-1\}.$$

Furthermore, this set is irredundant.

The notation is explained by the following.

(a) pt the point, with obvious $\mathbf{Z}/p$ action.

(b) $\mathbf{Z}/p$, p points with obvious $\mathbf{Z}/p$ action.

(c) $\mathbf{C}P_j^1$, $((p+1)/2 \leqslant j < p-1)$, the complex projective, line $\mathbf{C}P^1$ with $\mathbf{Z}/p$ action given by $[z_0; z_1] \longmapsto [z_0; \xi^j z_1]$ where $\xi = \exp(2\pi i/p)$.

(d) S_j, $(1 \leqslant j \leqslant (p-1/2)$, the Riemann surface of genus $(q-1)(p-1)/2$ associated to the complex function $u = (z^p - 1)^{1/q}$ where q satisfies $qj = -1 \bmod p$, $0 < q < p$. The action of $\mathbf{Z}/p$ on S_j is induced by $z \longmapsto \xi z$.

(e) C_j, $(1 < j \leqslant (p+1)/2)$, the complex projective plane $\mathbf{C}P^2$ with $\mathbf{Z}/p$ action given by $[z_0; z_1; z_2] \longmapsto [z_0; \xi z_1; \xi^j z_2]$.

(f) $\mathbf{C}P_j^n$ $(n \geqslant 2, 1 \leqslant j \leqslant p-1)$, the complex projective space $\mathbf{C}P^n$ with $\mathbf{Z}/p$ action given by $[z_0; z_1; \ldots; z_{n-1}; z_n] \longmapsto [z_0; z_1; \ldots; z_{n-1}; \xi^j z_n]$.

(g) Let M be a unitary $\mathbf{Z}/p$ manifold for which the $\mathbf{Z}/p$ action extends to a unitary S^1 action. For example, the unitary $\mathbf{Z}/p$ manifolds in (a), (c), (e) and (f) satisfy this property.

The circle S^1 acts freely on the product $M \times S^3$ by $(m, z_1, z_2) \longmapsto (tm, tz_1, tz_2)$, $t \in S^1$, where $S^3 = \{(z_1, z_2) \in \mathbf{C}^2; |z_1|^2 + |z_2|^2 = 1\}$. Let $\Gamma(M)$ denote the quotient $(M \times S^3)/S^1$, with $\mathbf{Z}/p$ action given by $[m, z_1, z_2] \longmapsto [\xi m, \xi z_1, z_2]$. Of course, this action extends to an S^1 action and we can define,

AMS (MOS) subject classifications (1970). Primary 57D85, 57E25; Secondary 57D20.
Key words and phrases. Equivariant bordism, $\mathbf{Z}/p$ manifolds, characteristic numbers.