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GENERALIZED STEENROD HOMOLOGY THEORIES

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Introduction. Alexander duality states that for a compactum X in S^n, $\check{H}^p(X) \cong H^c_{n-p-1}(S^n \backslash X)$, where $\check{H}^*$ is Čech cohomology and H^c_* is ordinary homology with compact supports.

In [St] Steenrod introduced a homology theory ${}^SH_*$ defined on the category CM of compact metric spaces with ${}^SH_p(X) \cong H^{n-p-1}(S^n \backslash X)$. ${}^SH_*$ is called *ordinary Steenrod homology*. Milnor [Mil] (see [K – S]) showed that ${}^SH_*$ is characterized by the Eilenberg-Steenrod axioms plus two additional axioms: invariance under relative homeomorphism; and a modified form of the continuity axiom in the form of a short exact sequence relating ${}^SH_*$ and $\check{H}_*$. In their recent work on extensions of C_*-algebras, Brown, Douglas and Fillmore [B – D – F] defined a functor Ext( ) on compacta whose restriction to finite complexes is K-homology. Kaminker and Schochet [K – S] axiomatized generalized Steenrod homology theories and showed that Ext( ) defines a generalized Steenrod homology theory on CM.

At the Operator Theory and Topology Conference (University of Georgia, April, 1975) Atiyah asked whether any generalized homology theory h_* defined on finite complexes admits a Steenrod extension to CM which preserves products and operations. We announce such an extension in Theorem 2, below. J. Kaminker recently informed us of an entirely different definition of generalized Steenrod homology theories due to D. S. Kahn, C. Schochet and himself.

Background and theorems. We will need an appropriate category and homotopy category of inverse systems of spaces. In [Gro] Grothendieck showed how to associate to any category C another category pro-C whose objects are inverse systems in C indexed by "filtering categories" and whose morphisms are so defined as to make cofinal systems isomorphic. In [Q] Quillen introduced the notion of a model category as an axiomatization of homotopy theory on Top and SS (= the category of simplical sets).

A model category is a category C together with three classes of morphisms called cofibrations, fibrations, and weak equivalences, which satisfy "the usual

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