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COMPONENT PROPERTIES OF SECOND ORDER LINEAR SYSTEMS

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Consider the second order linear system

$$x'' + A(t)x = 0, \tag{1}$$

where $A(t)$ is an n-by-n continuous matrix. Disconjugacy and other Sturm-type properties of solutions of (1) have been studied by a number of people (see e.g. [1]–[4]). Virtually no study has been made of the sign properties of the individual components of solutions of (1). It appears that such a study would be of interest not only from a theoretical point of view but also from a more practical point of view. We announce some results along this line, which are obtained under certain conditions on the matrix $A(t)$. For definitions and basic concepts one might consult [1] and [4].

THEOREM 1. *Let $A(t) = (a_{ij}(t))$ be symmetric with $a_{ij}(t) \geqslant 0$ whenever $i \neq j$ and $t \in [a, b]$, where b is the first conjugate point of a. Then there exists a nontrivial solution $u(t) = \operatorname{col}(u_1, \ldots, u_n)$ of (1) with $u(a) = u(b) = 0$, and $u_k(t) \geqslant 0$ on $[a, b]$, $k = 1, \ldots, n$.*

The converse of Theorem 1 holds in the following sense.

THEOREM 2. *Assume that $A(t) = (a_{ij}(t))$ is positive definite on (a, b) except at isolated points. If $a_{ij}(t) > 0$ on (a, b), and if there exists a nontrivial solution $y(t) = \operatorname{col}(y_1, \ldots, y_n)$ of (1) with $y(a) = y(b) = 0$ and $y_i(t) \geqslant 0$, $i = 1, \ldots, n$, on (a, b), then b is the first conjugate point of a.*

We recall that an n-by-n matrix $A = (a_{ij})$ is called irreducible if it is impossible to have $\{1, 2, \ldots, n\} = I \cup J$, $I \cap J = \varnothing$, $I \neq \varnothing \neq J$, and $a_{ij} = 0$ for all $i \in I$, $j \in J$.

THEOREM 3. *Let $A(t) = (a_{ij}(t))$ such that $a_{ij}(t)) \geqslant 0$ on $[a, b]$ and $A(t_0)$ is irreducible for some $t_0 \in (a, b)$. If $y(t) = \operatorname{col}(y_1, \ldots, y_n)$ is a nontrivial solution of (1) such that $y(a) = y(b) = 0$ and $y_i(t) \geqslant 0$ on (a, b), $i = 1, \ldots, n$, then $y_i'(a) > 0$, $y_i'(b) < 0$ and $y_i(t) > 0$ on (a, b), $i = 1, \ldots, n$. Moreover, if $w(t)$ is any solution of (1) with $w(a) = w(b) = 0$, then $w(t) = \alpha y(t)$ for some constant α.*

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