BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 2, March 1976

COMMUTATIVITY OF INTERTWINING OPERATORS. II

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Communicated by James Bramble, September 24, 1975

In a series of papers written partly in collaboration with E. M. Stein, the author has investigated the question of reducibility for the major unitary representations of a connected semisimple Lie group G of matrices. These representations are induced representations of a particular kind from a cuspidal parabolic subgroup and are the ones that appear in the Plancherel formula for G (see [1]). They split into at most finitely many irreducible pieces. One theorem in [7] is a dimension formula for the commuting algebra of such a representation. We can now give the full algebra structure of the commuting algebra, finding in particular that the irreducible constituents of each of the representations are inequivalent.

Let MAN be the Langlands decomposition of a parabolic subgroup of G; the subgroup is cuspidal if M has a discrete series. In this case let ξ be a discrete series representation of M, let λ be a unitary character of A, and let $U(\xi, \lambda, g)$ be the unitary representation of G induced from the representation of MAN given by $man \longrightarrow \lambda(a)\xi(m)$. The collection of all the $U(\xi, \lambda, \cdot\,)$ is the set of representations that we consider. Let $\mathcal{C}(\xi, \lambda)$ be the commuting algebra of $U(\xi, \lambda, \cdot\,)$.

THEOREM. *Let G be a connected semisimple Lie group of matrices, MAN a cuspidal parabolic subgroup, ξ a discrete series of M, and λ a unitary character of A. Then the commuting algebra $\mathcal{C}(\xi, \lambda)$ of $U(\xi, \lambda, \cdot\,)$ has a linear basis consisting of a set of unitary operators that form a group isomorphic to a direct sum of r copies of $\mathbf{Z}_2$ where $r \leqslant \dim A$. In particular, $\mathcal{C}(\xi, \lambda)$ is commutative.*

This theorem was obtained earlier [3], [4] for the case that MAN is a minimal parabolic subgroup, i.e., one with A as large as possible. The proof in the general case is considerably more complicated but does have points of contact with the proof in the minimal case. The main distinction is that M is generated by its identity component and center in the minimal case but not in the general case.

The operators that form the linear basis in the theorem are of a particular form; namely they are the intertwining operators listed in Theorem 2(ii) of [7]. The number r is easily computed, provided one knows some place where ξ can be imbedded infinitesimally in the nonunitary principal series of M; this proviso

AMS (MOS) subject classifications (1970). Primary 22E30, 22E45.
[1] Supported by NSF grant MPS75-07024.