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A FUNCTIONAL CALCULUS FOR SUBNORMAL OPERATORS

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ABSTRACT. In this announcement some results concerning a functional calculus for subnormal operators, in which the functions involved are not necessarily continuous, are presented. This functional calculus can be used to study the structure of subnormal operators.

Throughout this paper S is a subnormal operator on a separable Hilbert space $\mathcal{H}$, N is its minimal normal extension on $\mathcal{K}$, and μ is a scalar spectral measure for N. If T is any operator, $\mathcal{A}(T)$ is the ultraweakly closed algebra generated by T and 1, $\mathcal{W}(T)$ is the weak closure of $\mathcal{A}(T)$, and $\mathcal{W}^*(T)$ is the von Neumann algebra generated by T.

If $P^\infty(\mu)$ denotes the weak star closure of the polynomials in $L^\infty(\mu)$ then it is easy to show that $\mathcal{A}(N) = \mathcal{W}(N) = \{f(N): f \in P^\infty(\mu)\}$, and for every f in $P^\infty(\mu)$, $f(N)$ leaves $\mathcal{H}$ invariant. Define $f(S)$ by the formula $f(S)x = f(N)x$ for each x in $\mathcal{H}$; equivalently, $f(S) = f(N)|\mathcal{H}$.

THEOREM 1. $\mathcal{A}(S) = \{f(S): f \in P^\infty(\mu)\}$.

In [1, p. 89] Bram showed that if $T \in \mathcal{W}(S)$ then there is a unique R in $\mathcal{W}^*(N)$ such that $\|R\| = \|T\|$, $R\mathcal{H} \subseteq \mathcal{H}$, and $T = R|\mathcal{H}$. He also asked if this could be strengthened to get that $\mathcal{W}(S) = \{R|\mathcal{H}: R \in \mathcal{W}(N)\}$. Theorem 1 shows this to be possible if and only if $\mathcal{W}(S) = \mathcal{A}(S)$.

The next result has several applications in the study of subnormal operators.

THEOREM 2. *If (X, Ω, ν) is a finite measure space and $\mathcal{A}$ is any weak star closed subalgebra of $L^\infty(\nu)$ that contains* 1, *then there is a countable measurable partition $\{\Delta_0, \Delta_1, \ldots\}$ of X with the following properties*: (a) $\chi_{\Delta_n} \in \mathcal{A}$ *for all* $n \geq 0$; (b) *for $n \geq 1$, $\mathcal{A}_n \equiv \mathcal{A}\chi_{\Delta_n}$ is antisymmetric (i.e., every hermitian element in $\mathcal{A}_n$ is a multiple of χ_{Δ_n})*; (c) *$\mathcal{A}_0 \equiv \mathcal{A}\chi_{\Delta_0}$ is pseudosymmetric (i.e., for every subset Δ of Δ_0 with $\mu(\Delta) > 0$ there is an element of $\mathcal{A}_0$ that is real-valued on Δ but not constant there).*

Using the methods of [2] (and, in particular, Theorem 1 of [2]), the following is obtained.

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[2]Theorem 4 of this announcement appears in the second author's Ph.D. dissertation written at Indiana University under the supervision of John B. Conway. After completion of Olin's thesis the authors jointly studied antisymmetry for subalgebras of $L^\infty(\mu)$. The results of this work led to the remaining theorems in this paper.