BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 1, January 1976

THE DIRICHLET PROBLEM FOR A COMPLEX MONGE-AMPERE EQUATION

BY ERIC BEDFORD[1] AND B. A. TAYLOR[2]

Communicated by P. R. Halmos, September 25, 1975

On $\mathbf{C}^n$, write $d = \partial + \bar{\partial}$, $d^c = i(\bar{\partial} - \partial)$ so that $dd^c u = 2i\partial\bar{\partial}u$, and let

$$\beta_n = \left(\frac{i}{2}\right)^n \prod_{j=1}^n dz_j \wedge d\bar{z}_j$$

be the usual volume form. We study here the nonlinear Dirichlet problem,

$$\begin{aligned} &(dd^c u)^n = dd^c u \wedge \cdots \wedge dd^c u = f\beta_n \quad \text{on } \Omega, \\ (1) \qquad &u \text{ plurisubharmonic} \quad \text{on } \Omega, \\ &u = \phi \text{ on } \partial\Omega \end{aligned}$$

where Ω is a bounded open set in $\mathbf{C}^n$, $f \geqslant 0$, and ϕ is a continuous function on $\partial\Omega$. For arbitrary plurisubharmonic functions u, it is known that $dd^c u$ is a positive current of type (1, 1) [4, p. 70]; but, it is not clear that the higher exterior powers of $dd^c u$ are well defined. In fact, examples indicate that it is probably not possible to define $(dd^c u)^n$ as a distribution for all plurisubharmonic functions u [7]. However, for bounded, C^2 plurisubharmonic functions, Chern, Levine, and Nirenberg [3] have given an estimate which makes it clear how to define $(dd^c u)^n$ when u is a continuous plurisubharmonic function. If $\|u\|_\Omega = \sup\{|u(z)|: z \in \Omega\}$, then they prove that for each compact subset K of Ω, there is a constant $C = C(K)$ such that

$$\int_K (dd^c u)^n \leqslant C\{\|u\|_\Omega\}^n$$

for all C^2 plurisubharmonic functions u on Ω. With this result (and its proof), it is easy to show that the operator $(dd^c u)^n$, thought of as a mapping from the C^2 plurisubharmonic functions on Ω to the space of nonnegative Borel measures on Ω, has a continuous extension to the space of all continuous plurisubharmonic functions on Ω. It is with this definition of $(dd^c u)^n$ as a nonnegative Borel measure on Ω that we study the Dirichlet problem (1).

AMS (MOS) subject classifications (1970). Primary 32F05, 35D05; Secondary 32E99.

[1] Research supported in part by a Sloan Foundation Grant to Courant Institute of Mathematical Sciences, New York University, and by the Army Research Office grant number DAHC04-75-G-0149.

[2] Research supported in part by the National Science Foundation Grant Number GP 37628.