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VERSAL UNFOLDINGS OF G-INVARIANT FUNCTIONS

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1. We announce here some results on equivariant local differential analysis. The proofs will appear elsewhere [7]. We consider a compact Lie group G, acting orthogonally on R^n. $C^\infty(x)$ (respectively $C^\infty(R^n)$) will denote the ring of germs of C^∞ functions around $0 \in R^n$ (the ring of C^∞ functions of R^n). The germ of R^n at 0 will be denoted by X. $C^\infty(x)^G$, $C^\infty(R^n)^G$ will denote the G-invariant germs (functions). We shall consider parameter (germs of) spaces $U, V, \ldots,$ on which G acts, by definition, trivially.

If $f(x) \in C^\infty(x)^G$, an *unfolding* of $f(x)$ is an $F(x, u) \in C^\infty(x, u)^G$ such that $F(x, 0) \equiv f(x)$. The unfolding $F(x, u)$ is *versal*, if any other unfolding of $f(x)$, $H(x, v) \in C^\infty(x, v)^G$, can be induced from F, by a commutative diagram

$$\begin{array}{ccc} X \times V & \xrightarrow{\Phi} & X \times U \\ \downarrow & & \downarrow \\ V & \xrightarrow{\varphi} & U \end{array}$$

such that:

(a) $\Phi, \varphi \in C^\infty$,

(b) Φ is G-equivariant,

(c) $\Phi | X \times 0 \equiv \mathrm{id}\, X$,

(d) $H = F \circ \Phi$.

G also acts on smooth vector-fields on $X(R^n)$. We consider the *invariant* (germs of) vector-fields $\Gamma^\infty(TX)^G \subset \Gamma^\infty(TX)$ i.e., fields such that $g\xi(x) = Tg(\xi(x)) = \xi(gx)$. $\Gamma^\infty(TX)^G$ is a $C^\infty(x)^G$-module moreover, if $f(x) \in C^\infty(x)^G$, the subset

$$J_G(f) = \{df(\xi), \xi \in \Gamma^\infty(TX)^G\} \subset C^\infty(x)^G.$$

is an ideal, called the *G-jacobian ideal of f*. We shall assume that f is given, and that $\dim_R C^\infty(x)^G/J_G(f) < \infty$.

By definition $F(x, u) \in C^\infty(x, u)^G$, unfolding of f, is *infinitesimally versal* if the images of $\partial F(x, 0)/\partial u_1, \ldots, \partial F(x, 0)/\partial u_k$ in $C^\infty(x)^G/J_G(f)$ generate the R-vector space $C^\infty(x)^G/J_G(f)$.

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