BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 1, January 1976

A SUFFICIENT CONDITION FOR k-PATH HAMILTONIAN DIGRAPHS

BY JOHN ROBERTS

Communicated by Walter Gautschi, September 7, 1975

A directed graph (or digraph) D is: (1) *traceable* if D has a hamiltonian path; (2) *hamiltonian* if D has a hamiltonian cycle; (3) *strongly hamiltonian* if D has arcs and each arc lies on a hamiltonian cycle; (4) *hamiltonian-connected* if D has a hamiltonian u-v path for every pair of distinct vertices u and v; (5) *k-path traceable* if every path of length not exceeding k is contained in a hamiltonian path; and (6) *k-path hamiltonian* if every path of length not exceeding k is contained in a hamiltonian cycle.

The indegree and the outdegree of a vertex v are denoted by $\mathrm{id}(v)$ and $\mathrm{od}(v)$ respectively. A digraph D of order p is of *Ore-type* (k) if $\mathrm{od}(u) + \mathrm{id}(v) \geqslant p + k$ whenever u and v are distinct vertices for which uv is not an arc of D.

In this research announcement we outline a proof of the following result, a complete proof of which will appear elsewhere, and present some consequences of it.

THEOREM. *If a nontrivial digraph D is of Ore-type* (k), $k \geqslant 0$, *then D is k-path hamiltonian.*

PROOF. Let D have order $p \geqslant 2$. First, observe that D is strong. Since the result holds if D is the complete symmetric digraph K_p, we assume that $D \neq K_p$. This in turn implies that $p \geqslant k + 4$. Also, it can be shown that every path of length not exceeding k is contained in a path of length $(k + 1)$ and this longer path is contained in a cycle.

Suppose D has a path P: $v_1, v_2, \ldots, v_{k+1}$ of length k which is contained in no hamiltonian cycle. Let C: $v_1, v_2, \ldots, v_n, v_1$ be any longest cycle containing P. Then, $V \equiv V(D) - V(C) \neq \varnothing$, where $V(D)$ and $V(C)$ denote the vertex sets of D and C respectively.

Now, assume that V has distinct vertices u and v for which $uv \notin E(D)$ and the subdigraph $\langle V \rangle$ induced by V has no v-u path. Then, $vu \notin E(D)$ implies that

$$(1) \qquad p + k \leqslant \mathrm{od}(v) + \mathrm{id}(u) \leqslant p - n - 2 + \mathrm{od}(v, C) + \mathrm{id}(u, C)$$

where $\mathrm{od}(v, C)$ and $\mathrm{id}(u, C)$ denote the number of vertices in C which are

AMS (MOS) subject classifications (1970). Primary 05C20.

Key words and phrases. Digraphs, traceable, hamiltonian, hamiltonian-connected, strongly hamiltonian, k-path hamiltonian, k-path traceable.