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OPTIMAL LIPSCHITZ AND L^p ESTIMATES FOR THE EQUATION $\bar\partial u = f$ ON STRONGLY PSEUDO-CONVEX DOMAINS[1]

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For definitions and notation in what follows, see Hörmander [5]. Let $\mathcal{D} \subset\subset \mathbf{C}^n$ be strongly pseudo-convex with C^5 boundary. Let

$$\Lambda_\alpha(\mathcal{D}) = \left\{ f: \mathcal{D} \rightarrow \mathbf{C} : \|f\|_{L^\infty} + \sup_{z, z+h \in \mathcal{D}} \frac{|f(z) - f(z+h)|}{|h|^\alpha} = \|f\|_{\Lambda_\alpha} < \infty \right\},$$

$$L^p(\mathcal{D}) = \left\{ f: \mathcal{D} \rightarrow \mathbf{C} : \int_\mathcal{D} |f|^p \, dL < \infty \right\}, \qquad 1 \leqslant p < \infty,$$

where dL is Lebesgue measure.

We wish to announce Lipschitz and L^p regularity results for Henkin's solution to $\bar\partial u = f$, f a (0, 1) form with $\bar\partial f = 0$, which are essentially best possible, not only for his solution, but for any solution to the equation. More precisely,

THEOREM 1. *There exists a linear operator T taking $\bar\partial$ closed* (0, 1) *forms with coefficients in $C^\infty(\mathcal{D})$ to functions in $C^\infty(\mathcal{D})$ and satisfying*

(a) $\bar\partial Tf = f$,

(b) $\|Tf\|_{L^q} \leqslant A_p \|f\|_{L^p}$, $1 < p < 2n + 2$, $1/q = 1/p - 1/(2n + 2)$,

(c) $\|Tf\|_{\Lambda_{1/2-(n+1)/p}} \leqslant A_p \|f\|_{L^p}$, $2n + 2 < p \leqslant \infty$,

(d) $\|Tf\|_{L^{(2n+2)/(2n+1)-\epsilon}} \leqslant A_\epsilon \|f\|_{L^1}$, $\epsilon > 0$,

(e) $\int_\mathcal{D} \exp(a/\|f\|_{L^{2n+2}} |Tf|^{(2n+2)/(2n+1)}) \, dL \leqslant C$, *where a, C do not depend on f.*

The constants a, C, A_ϵ, A_p are independent of "small" perturbations of $d\mathcal{D}$.

We give examples to show that

(b′) $\exists \mathcal{D} \subset\subset \mathbf{C}^n$ and $f_p \in C^\infty_{(0,1)}(\mathcal{D})$ such that $\mathcal{D}$ is strongly pseudo-convex, $\|f_p\|_{L^{p-\epsilon}} < \infty$ $\forall \epsilon > 0$, $\bar\partial f_p = 0$, and no u satisfies both $\bar\partial u = f_p$ and $\|u\|_{L^q} < \infty$, $1/q = 1/p - 1/(2n + 2)$, $1 < p < 2n + 2$.

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