

TOPOLOGY AND LOGIC AS A SOURCE OF ALGEBRA

*Retiring Presidential Address*¹

BY SAUNDERS MAC LANE

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Introduction. Each President of the American Mathematical Society is required to present a retiring presidential address. By custom, this address is not in any way directed to the examination of the administrative or professional problems of the Society. This is not because these problems are unimportant, but is presumably because the President's contribution to them should have long since been made, or not made, as the case may be, well before his retirement from office. Instead the custom is that the President's address is concerned with contributions to our science. This is a good custom, since it is the mathematical theorems and the solutions of mathematical problems which possess the quality of permanence and the symmetry of structure which is our primary objective in mathematics.

My own research work has been largely concerned with aspects of algebra—a variety of aspects by no means covering all of algebra, but chiefly involved in some explorations of the relation between algebra and the neighboring fields of logic and geometry. These studies have given me the lively impression that many of the ideas of algebra do indeed arise from these other fields, and that this origin highlights the sense in which the science of mathematics exemplifies the interdependence of its parts. Hence, this address will be devoted to an examination of certain of the ways in which the problems from geometry and logic arising in my own research work have illuminated algebra or contributed new concepts to that field.

1. Separable extensions. My first example is a problem in the theory of fields whose solution required the use of ideas from geometry—more exactly, ideas about an axiomatic treatment of linear independence.

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