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THE HOLOMORPHIC LEFSCHETZ FORMULA

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Let X be a compact complex manifold and let $f: X \longrightarrow X$ be a holomorphic map. One can assign to each component Y of the fixed point set F of f a complex number $\nu_Y(f)$ so that

$$L(f, 0) = \sum_Y \nu_Y(f).$$

$L(f, 0)$ denotes the Lefschetz number on $H^*(X, 0)$. In this note we outline a computation of $\nu_Y(f)$ in the case that Y is a *nondegenerate* component of F, i.e., Y is a *submanifold* of X, and if df^N denotes the map induced by df on the normal bundle of Y, then $\det(1 - df^N) \neq 0$. Our result is that $\nu_Y(f)$ is given by the same formula proved by Atiyah and Singer [3] in the case that f is an isometry. If $\dim Y = 0$ the formula was known without this restriction on f by Atiyah and Bott [2]. Patodi [7] was able to remove the restriction on f under other assumptions, which are vacuous if $\dim Y = 1$.

Our methods are purely algebraic, and go through in algebraic geometry of characteristic zero. In particular, for $f =$ identity, we obtain a simpler justification of the local formula used in [8] to prove the Riemann-Roch theorem that is also valid in the algebraic category.

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1. **Statement of the formula.** Let N denote the normal bundle of Y and let $\lambda_1, \ldots, \lambda_m$ be the eigenvalues of df^N. N splits as direct sum of bundles N_i of dimension d_i on which $df^N - \lambda_i 1$ is nilpotent. Then the component of degree zero of the characteristic class $\Sigma_{p=0}^{d_i} (-1)^p \lambda_i^p \operatorname{ch}(\Lambda^p N_i^*)$ is $(1 - \lambda_i)^{d_i} \neq 0$, hence this class is invertible in the cohomology ring of Y. The formula for $\nu_Y(f)$ is

$$\nu_Y(f) = \int_Y \mathcal{T}(Y) \left\{ \prod_{i=1}^{m} \sum_{p=0}^{d_i} (-1)^p \lambda_i^p \operatorname{ch}(\Lambda^p N_i^*) \right\}^{-1} \tag{1.1}$$

$\mathcal{T}(Y)$ is the total Todd class of Y and the integral sign denotes evaluation on the fundamental cycle. We always think of characteristic classes as taking values in

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