INVARIANCE PRINCIPLE FOR MODIFIED WAVE OPERATORS

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1. **Introduction.** The invariance principle of Birman and Kato (see e.g. [5])
states that, for simple scattering systems with short-range potentials, the wave op-
erator limits

$$(1) \qquad \Omega_{\pm}(\phi(H_2),\ \phi(H_1)) \equiv \underset{t \longrightarrow \pm \infty}{\text{s-lim}}\ e^{it\phi(H_2)}e^{-it\phi(H_1)}P_1$$

are independent of ϕ for a wide class of functions, and equal, respectively, the
wave operators

$$(2) \qquad \Omega_{\pm}(H_2,H_1) \equiv \underset{t \longrightarrow \pm \infty}{\text{s-lim}}\ e^{itH_2}e^{-itH_1}P_1 .$$

Kato first proved the invariance under the assumption that $H_2 - H_1$ is a trace-
class operator. It has since been proved under some alternative assumptions on
H_1 and H_2 (see e.g. [4], [6]).

For other scattering systems, such as scattering with long-range potentials,
the limits (1) and (2) may not exist; however, certain modified wave operators
(see (3) and (4) below) may exist [1], [3]. An invariance principle for modified
wave operators has been proved by Matveev [6], [7] and Sakhnovich [8] under
certain rate-of-convergence assumptions. However, these assumptions are shown
to be satisfied only for a class of short-range potentials [6, Theorem 2].

In this note we announce the result that the invariance principle of scatter-
ing theory is valid in practically all situations in which (possibly modified) time-
dependent wave operators are known to exist.

2. **Notation.** Let H_k be selfadjoint operators on separable Hilbert spaces
H_k, $k = 1, 2$. Let P_1 denote the orthogonal projection of H_1 onto the space
$H_{1,ac}$ of absolute continuity for H_1. Let Δ be some closed and bounded interval
of the real axis $\mathbf{R}$, and let $E_1(\Delta)$ be the corresponding spectral projection of the
operator H_1. Let $\mathcal{D}_1$ be the dense subset of vectors $u \in H_{1,ac}$ with $\|u\| < \infty$,