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SOME INVARIANTS OF GENERIC IMMERSIONS AND THEIR GEOMETRIC APPLICATIONS

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1. This is an announcement of some theorems to appear in full detail elsewhere [2].

Let X, Y be smooth manifolds, with $\dim X < \dim Y$ and $f\colon X \longrightarrow Y$ a *generic* immersion. To f we will attach two numerical invariants $\mu_2(f)$ and $\nu_3(f)$, which will be described in the next paragraphs; it is conceivable that a more cohomological, characteristic-classes-type approach to these invariants should be possible. Anyway, granted their definition one has the following results:

THEOREM. *Let $f\colon X \longrightarrow Y$ be a generic immersion as above. The necessary and sufficient condition for the existence of a smooth embedding $X \xrightarrow{F} Y \times R$ lifting f is that $\mu_2(f) = \nu_3(f) = 0$.* □

F "lifts f" means that the following diagram is commutative:

$$\begin{array}{ccc} & & Y \times R \\ & \overset{F}{\nearrow} & \downarrow \\ X & \xrightarrow[f]{} & Y \end{array}$$

COROLLARY 1. *Suppose that $\pi_1 X = 0$. The necessary and sufficient condition for the existence of a smooth embedding $X \xrightarrow{G} Y \times S_1$ lifting f is that $\mu_2(f) = \nu_3(f) = 0$.* □

The next corollary has some connection with the group Θ_3 of Milnor and Kervaire [1]. We consider a smooth homotopy 3-sphere Σ_3 and two points p_0, $p_1 \in \Sigma_3$ $(p_0 \neq p_1)$. We consider two small 2-spheres, in Σ_3, of centers p_0 and p_1: S_2^0, S_2^1. By the Smale-Hirsch immersion theory there is a (generic) regular homotopy:

$$f \in \operatorname{Imm}_I (S_2 \times I, (\Sigma_3 - \{p_0, p_1\}) \times I)$$

connecting S_2^0, S_2^1. (The subscript I means that f is level-preserving.)

COROLLARY 2. *Let Σ_3 be a smooth homotopy 3-sphere, and f some gener-*

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