Elementary induction on abstract structures, by Yiannis Nicholas Moschovakis, Studies in Logic and the Foundations of Mathematics, vol. 77, North-Holland, Amsterdam; American Elsevier, London, New York, x+218 pp., $17.75.

This review is divided into three parts. In the first section we define, by means of examples, the objects studied in the book under review. The second section discusses the book itself. The third section is more technical. It discusses a way that the second-order assumption of "acceptability" imposed on structures in Chapter 5 can be weakened to make the theory developed in Chapters 5–8 relevant to ordinary first-order model theory.

Inductive definitions. The way to tell a logician from his mathematical colleague is by his attention to the language of mathematics. The logician takes as a fundamental tenet that light can be shed on mathematical problems by simply paying attention to, and then analyzing, the language in which mathematics is formulated and carried out. This book presents a detailed analysis of one part of the language of mathematics, namely inductive (to be precise, first-order positive inductive) definitions on a fixed structure $\mathfrak{A}$.

An *inductive definition* can be viewed as a monotone operator Γ on sets, monotone in the sense that $X \subseteq Y$ implies $\Gamma(X) \subseteq \Gamma(Y)$. It has associated with it certain *stages* $I_\Gamma^0 \subseteq I_\Gamma^1 \subseteq \cdots \subseteq I_\Gamma^\alpha \cdots$, a smallest *fixed point* I_Γ, and a *closure ordinal* $\|\Gamma\|$, equal to the least ordinal number β such that $I_\Gamma = \bigcup_{\alpha<\beta} I_\Gamma^\alpha$.

A. ERDÖS NUMBERS. We begin with a slightly frivolous example which shows that inductive definitions arise in real life. Let M be the set of mathematicians with a distinguished element $e \in M$. For $X \subseteq M$, let $\Gamma(X)$ be the set of those mathematicians who have published a joint paper p and one of the authors of p is in X. Let $I^0 = \{e\}$, $I^{n+1} = \Gamma(I^n)$ and let I_Γ be the union of the various I^n. If e is properly chosen, then Γ is an inductive definition of the set of mathematicians that have Erdös numbers,[1] and I^n is the set of mathematicians with Erdös number $\leqq n$. Notice that I_Γ is a fixed point of Γ, $\Gamma(I_\Gamma) = I_\Gamma$, even if there are an infinite number of authors of some paper. Thus the closure ordinal is at most ω, the first infinite ordinal.

B. A MATE FOR WHITE IN α MOVES. Consider some two-person game like chess. Let W be the set of positions from which white has a winning strategy. We can give a more informative inductive definition of this set as follows. For any set X of positions, let $\Gamma(X)$ be the set of all positions such that, for any move of black, white has a response putting him in X. Let I^0 be the set of positions which are mates for white and let I^{n+1} be $\Gamma(I^n)$. Thus I^n is the set of positions which are mates for white in n moves. In ordinary chess, where each player has a finite number of possible moves at any one turn, W is simply the union of the various I^n, and W is the smallest fixed

[1] The notion of Erdös number was defined in *What is your Erdös number?* by C. Goffman, Amer. Math. Monthly **76** (1969), 791.