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A GEOMETRIC PROOF OF THE STRONG MAXIMAL THEOREM

BY A. CORDOBA AND R. FEFFERMAN

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In R^n, suppose we consider the operator M_s given by

$$M_s(f)(x) = \sup_R \frac{1}{|R|} \int_R |f(y)|\, dy$$

where f is a locally integrable function on R^n and the sup is taken over all rectangles with sides parallel to the axes which contain the point x. Then the strong maximal theorem may be taken as the statement that M_s is bounded from $L(\log^+ L + 1)^{n-1}(Q)$ to weak $L^1(Q)$, i.e.

$$m\{M_s f > \alpha\} \leqslant A_n \int \frac{|f(x)|}{\alpha} \log^{n-1}\left(\frac{|f(x)|}{\alpha} + 1\right) dx$$

where A_n is some absolute constant, and Q is the unit cube in R^n.

Our result consists of a purely geometric argument establishing such an inequality. At the heart of the matter is a geometric proof of the following covering lemma:

Suppose $R_1, R_2, \ldots, R_k, \ldots$ is a sequence of rectangles contained inside the unit cube in R^n. Then there is a subcollection $\widetilde{R}_1, \widetilde{R}_2, \ldots$ of the R_k's satisfying the following conditions:

(1) $|\bigcup \widetilde{R}_k| \geqslant c_n |\bigcup R_k|$ for some absolute constant $c_n > 0$, and

(2) $\|\exp(\Sigma\, \chi_{\widetilde{R}_k})^{1/(n-1)}\|_{L^1} \leqslant C_n |\bigcup R_k|$ for some absolute constant $C_n < \infty$.

These observations lead to further results in the theory of differentiation of the integral.

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DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY, PRINCETON, NEW JERSEY 08540

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