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THE WALL OBSTRUCTION IN SHAPE AND PRO-HOMOTOPY, WITH APPLICATIONS

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1. **Geometrical results.** There exist (CW) complexes X homotopy dominated by finite complexes but not homotopy equivalent to finite complexes [8]. It is unknown if there are compact metric spaces (compacta) with this property. However, one may construct a compactum, Z, shape dominated by a finite complex but not shape equivalent (hence not homotopy equivalent) to a finite complex by the following trick. Let

$$X \underset{d}{\overset{u}{\rightleftarrows}} K$$

be a homotopy domination of X (above) by a finite complex K; $d \circ u$ is homotopic to 1. Form the inverse sequence

$$K \xleftarrow{u \circ d} K \xleftarrow{u \circ d} K \xleftarrow{u \circ d} \cdots .$$

This sequence is isomorphic to X in pro-homotopy [1]. Hence its inverse limit, Z, is a compactum shape equivalent to X: see [3]. Since homotopy theory and shape theory agree on complexes, Z has the required properties. By [8], K may be chosen two dimensional. So we will assume Z is two dimensional (and connected).

Embed Z in S^5 with $S^5\backslash Z$ 1-ULC. Then $S^5\backslash Z$ is not homeomorphic to the interior of a compact manifold: otherwise Z would be a shape deformation retract of a compact topological manifold neighborhood of Z in S^5, and such a neighborhood would have finite homotopy type. But, by Siebenmann's theory of I-regular neighborhoods [7], the end of $S^5\backslash Z$ is *tame*, in the sense of [6]. So $S^5\backslash Z$ has nonvanishing Siebenmann obstruction (a *strange* end [6]). So has $S^n\backslash Z, n > 5$.

The map $u \circ d$: $K \longrightarrow K$ is a homotopy idempotent, but it is not homotopic to a strict idempotent, *not even stably.* For, the inverse limit of the sequence obtained by repeating a strict idempotent is a (compact) ANR, and this compact ANR would be shape equivalent to Z, contradicting [9]. Details appear in [3].

2. **Shape and pro-homotopy.**

THEOREM 1. *Let Z be a connected compactum, $z \in Z$. The following are equivalent*: (i) Z *is shape dominated by a finite complex*; (ii) Z *is shape*

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