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CONVEXITY FOR A SIMPLY CONNECTED p-ADIC GROUP

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In [4] Kostant showed that the set of Iwasawa double cosets which intersect a given Cartan double coset in a semisimple Lie group corresponds to a certain convex subset in the Lie algebra of a maximal torus of the group. As a consequence, he established that representatives for the double cosets relative to a maximal compact subgroup of a semisimple Lie group may be chosen in the unipotent radical of a minimal parabolic subgroup. We announce here analogues of these results for a simply-connected p-adic group.

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Let $\mathbf{G}$ be a connected simply-connected semisimple algebraic group defined over a p-adic field Ω. Let G denote the group of Ω-rational points of $\mathbf{G}$. Then G is a locally compact totally disconnected group. Borel and Tits [1] and Bruhat and Tits [2] have shown that G has a structure theory which is similar in many ways to that of a semisimple Lie group.

Let $\mathbf{P}$ be a minimal parabolic Ω-subgroup of $\mathbf{G}$. Then $\mathbf{P}$ is a split product $\mathbf{P}$ $\mathbf{P} = \mathbf{MN}$ in which $\mathbf{M}$ is connected and reductive and $\mathbf{N}$ is the unipotent radical of $\mathbf{P}$. Let $\mathbf{A}$ be the maximal Ω-split torus in the center of $\mathbf{M}$. For simplicity, we denote the group of Ω-points of each of the above algebraic groups by the corresponding ordinary capital letter.

Let ${}^\circ A$ be the maximal compact subgroup of A. Then $A/{}^\circ A$ is a free $\mathbf{Z}$-module of rank equal to the Ω-rank of $\mathbf{G}$. We call $\mathfrak{a} = (A/{}^\circ A) \otimes_{\mathbf{Z}} \mathbf{R}$ the Lie algebra of A and write $H: A \longrightarrow \mathfrak{a}$ for the natural map which imbeds $A/{}^\circ A$ as a lattice in $\mathfrak{a}$. To any rational character χ of $\mathbf{A}$ we associate a linear functional on $\mathfrak{a}$ by setting $\log|\chi(a)| = \langle \chi, H(a) \rangle$ $(a \in A)$. The relative Weyl group $W = N_G(A)/Z_G(A)$ operates on A and $\mathfrak{a}$. There is a root system in the dual $\mathfrak{a}^*$ of $\mathfrak{a}$ associated to the restriction to $\mathbf{A}$ of the adjoint representation of $\mathbf{G}$. Choosing a W-invariant scalar product on $\mathfrak{a}$, we regard this root system as a subset of $\mathfrak{a}$. Let N correspond to a set of positive roots and let $\bar{N}$ be the Ω-points of the unipotent radical of the opposite parabolic subgroup, corresponding to the negative roots. Write $\mathfrak{a}^+$ [respectively, $+\mathfrak{a}$] to denote the (closed) positive chamber in $\mathfrak{a}$ [respectively, the cone consisting of nonnegative linear combinations of the positive roots]. The mapping H extends to M; denoting the kernel of H in M as ${}^\circ M$, we see that H also imbeds $M/{}^\circ M$ as a lattice in $\mathfrak{a}$. Let $M^+ = H^{-1}(\mathfrak{a}^+)$ and $+M = H^{-1}(+\mathfrak{a})$.

We choose a particularly "good" maximal compact subgroup K of G (i.e.

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