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ON THE NACHBIN TOPOLOGY IN SPACES OF HOLOMORPHIC FUNCTIONS

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1. **Introduction.** $H(U)$ denotes the vector space of all holomorphic functions on an open subset U of a complex Banach space E. In this note we announce results concerning the Nachbin topology τ_ω in $H(U)$. τ_ω is useful in the study of holomorphic continuation; see Dineen [5], [7] and Matos [8]. We recall the definition of τ_ω; see Nachbin [10]. A seminorm p on $H(U)$ is said to be ported by a compact subset K of U if for each open set V, with $K \subset V \subset U$, there exists $c(V) > 0$ such that $p(f) \leqslant c(V) \sup_{x \in V} |f(x)|$ for all $f \in H(U)$. The locally convex topology τ_ω is defined by all such seminorms. To study $(H(U), \tau_\omega)$ we consider the vector spaces of holomorphic germs $H(K)$ with $K \subset U$ compact. We endow each $H(K)$ with the inductive topology given by

$$H(K) = \varinjlim_{\epsilon > 0} H^\infty(K_\epsilon),$$

where $K_\epsilon = \{x \in E: \operatorname{dist}(x, K) < \epsilon\}$ and $H^\infty(K_\epsilon)$ denotes the Banach space of all bounded holomorphic functions on K, with the sup norm.

2.[1] **Completeness of** $(H(U), \tau_\omega)$. The following theorem answers a question raised by Nachbin [11].

THEOREM 1. *$(H(U), \tau_\omega)$ is always complete.*

Earlier partial results were given by Dineen [6], Chae [3] and Aron [2] for U "nice". We give an indication of the proof of Theorem 1. For each compact $K \subset U$, let M^K denote the image of the canonical mapping $H(U) \longrightarrow H(K)$. After identifying $H^\infty(K_\epsilon)$ with its image in $H(K)$, we define:

$$M_\epsilon^K = M^K \cap H^\infty(K_\epsilon),$$
$$\widetilde{M}_\epsilon^K = \text{closure of } M_\epsilon^K \text{ in } H^\infty(K_\epsilon),$$
$$\widetilde{M}^K = \bigcup_{\epsilon > 0} \widetilde{M}_\epsilon^K.$$

In a diagram we have

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[1] The results in §2 of this note are taken from the author's doctoral dissertation at the University of Rochester, written under the supervision of Professor Leopoldo Nachbin.